F08, G15 and few others. Thus, it can lead to SSS problem on the classical LDA that is why LDA/GSVD shall be used.

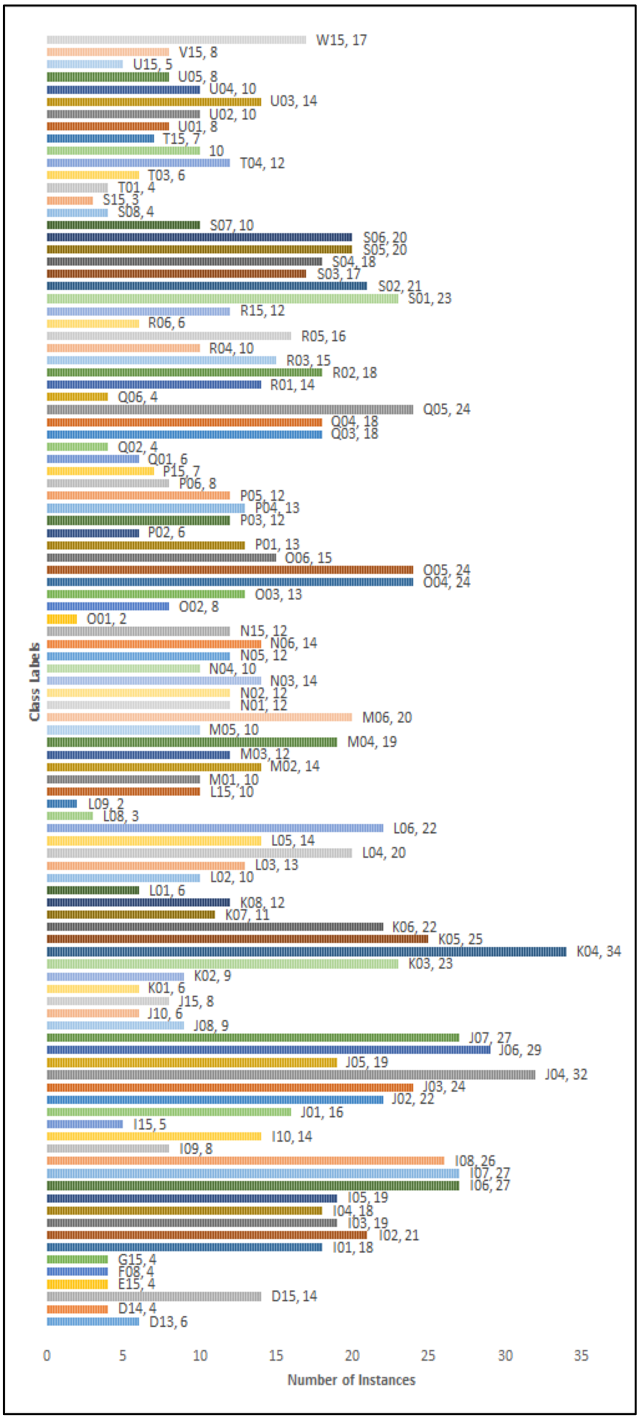

Fig. 1 Classes and number of instances of the BLE RSSI dataset

The study introduced a new approach for LDA/GSVD by utilizing ANN. The tansigmoid transmission function was utilized for the hidden layers' activation function. The flow of the procedure in training the enhanced algorithm is shown in figure 2, and the trained network's architecture is presented in figure 3. The ANN architecture is formed from 13 input variables which are the dimensions, and the corresponding 13 output variables are the expected feature subspaces. For the architecture to learn and predict the possible outcomes, the feature subspaces must be derived from the existing LDA/GSVD algorithm (table 2) since each dimension will have a corresponding feature subspace.

These dimensions and feature subspaces will be used in training and testing. For the sampling, 70% of the instances of the dataset were allocated for training, and 30% for the testing. Moreover, in training of the network, Bayesian Regularization Back Propagation (BRBP) was employed.

TABLE II
EXISTING LDA/GSVD ALGORITHM

Algorithm: **Existing LDA/GSVD Algorithm**
For the matrix $A \in R^{m\times n}$ with k groups, it calculates the matrix's columns $G \in R^{m\times(k-1)}$, which maintains the configured cluster dimensionally narrowed space, and determines $(k-1)$-dimensional depiction Y of A. Step 1: Calculate $H_w \in R^{m\times n}$ and $H_b \in R^{m\times k}$ from A Step 2: Solve the $K = (H_b, H_w)^T \in \mathrm{R}^{(k+n)\times m}$ for its orthogonal decomposition. $$P^T K Q = \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix}$$ Step 3: Let $t = \mathrm{rank}(K)$. Step 4: Calculate W from the SVD of $P(1:k, 1:t)$, which is $U^T P(1:k, 1:t)\ W = \Sigma_A$. Step 5: Solve the first $k-1$ columns of $$X = Q\begin{pmatrix} R^{-1}W & 0 \\ 0 & I \end{pmatrix}$$ and allocate those to G. Step 6: $Y = G^T A$.

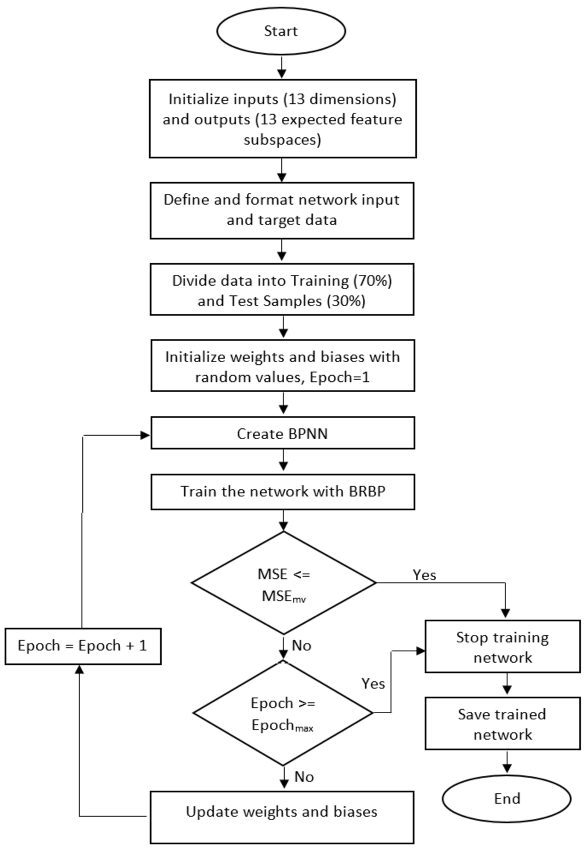

Fig. 2 Training process flowchart of the ANN for LDA/GSVD

After saving the trained network, it will become a module or subroutine that will be used to solve the expected new feature subspaces of the inputs. Thus, the algorithm (table 3) was used to compute the feature subspaces of the instances of the BLE RSSI dataset.

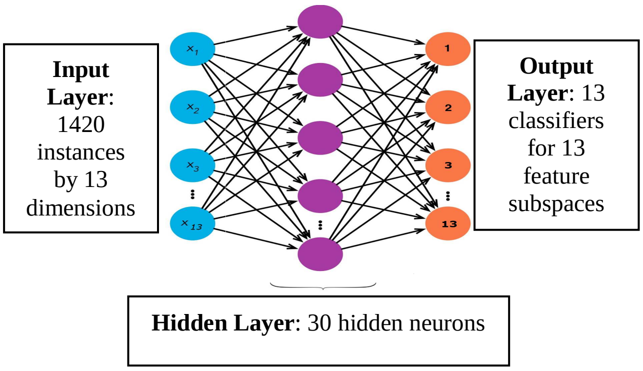

Fig. 3 Artificial Neural Network Architecture for the LDA/GSVD algorithm

TABLE III
COMPUTATION OF THE FEATURE SUBSPACES USING THE ENHANCED LDA/GSVD ALGORITHM

Algorithm: **Computation of the Feature Subspaces**
1. Enter the 13 values of the 13 dimensions.
2. Compute the 13 feature subspaces using the module from the trained network.
3. Return the computed feature subspaces.

III. Result and Discussion

Using MATLAB R2014a, both algorithms, existing and enhanced LDA/GSVD, were coded and ran on a PC with the processor of Intel® Core i5, 4GB RAM, and 2.7GHz speed.

A. Dataset without LDA/GSVD Classification

Figure 4 below shows the graphical representation of the data without performing LDA/GSVD. Since the dataset is multi-dimensional, for this example, only the first two dimensions are shown in the graph. It can be seen that instead of 105 classes, data points are grouped in approximately three (3) classes. Thus, all of these data points cannot be distinguished as to what classes they belong.

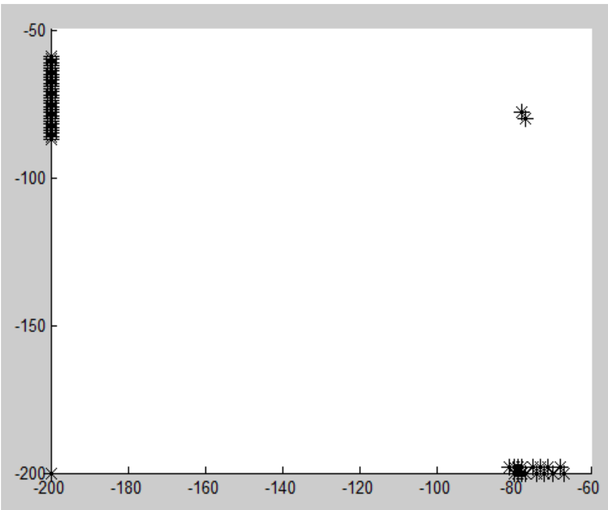

Fig. 4 Graph of the dataset without applying LDA/GSVD

B. Existing LDA/GSVD Algorithm

For classifying 105 classes with 13 dimensions, and a total of 1420 instances, the LDA/GSVD algorithm took 7 seconds to finish. The computational cost was obtained using equation 1.

$$CC = ET - ST \tag{1}$$

Where CC is the computational cost, ET means end time or the time when the program finished to execute all the instructions), and ST means start time or the time when the program starts to execute.

Figure 5 shows the graph for the feature subspaces of the first two dimensions of the dataset using the existing LDA/GSVD. Also, figure 5 shows a better separation of data points compared to figure 4. Due to the number classes and instances, most of the data points with same feature subspaces overlap with each other.

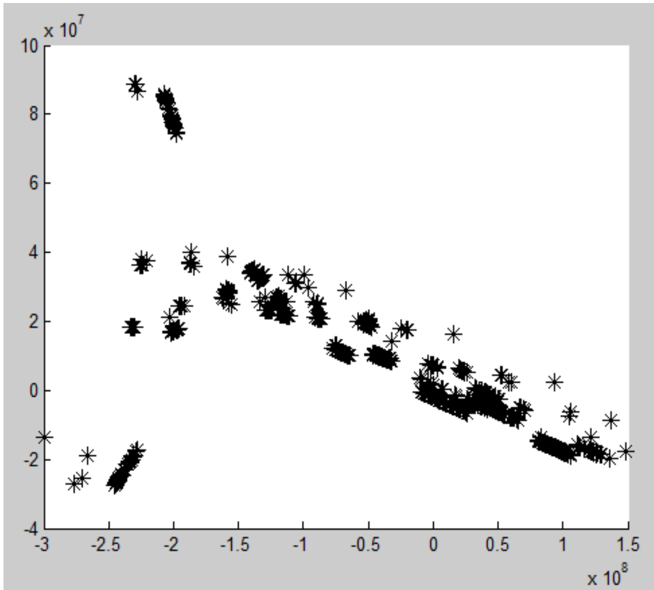

Fig. 5 Graph of the feature subspaces after applying LDA/GSVD

C. Enhanced LDA/GSVD

The performance functions were used in the study which includes the Mean Squared Error (MSE) and Regression (R) to evaluate the performance of the ANN for LDA/GSVD algorithm. MSE is the average squared difference between experimental output values and the fed targets in training.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(t_i - a_i)^2 \tag{2}$$

Where n is the sample set's size, a_i is the ANN experimental or observed output and t_i is the matching targets. Regression (R) computes the outputs and targets' correlation. When the value of R is 1, it signifies a good or close relationship, otherwise a random relationship [38].

Figure 6 depicts the performance of training and test samples using BRBP algorithm. The graph shows that the test and training samples overlap with each other. Thus, training and test curves continue to stabilize every time the epoch increments. At epoch 1500 the MSE error is approximately 2.5115 x10^{-3}. Further, the histogram in figure 7 presents the frequency of the instances per error. The measurement of the error is by subtracting the targets and the resultant outputs. The most significant error in the training was at around 0.2443.

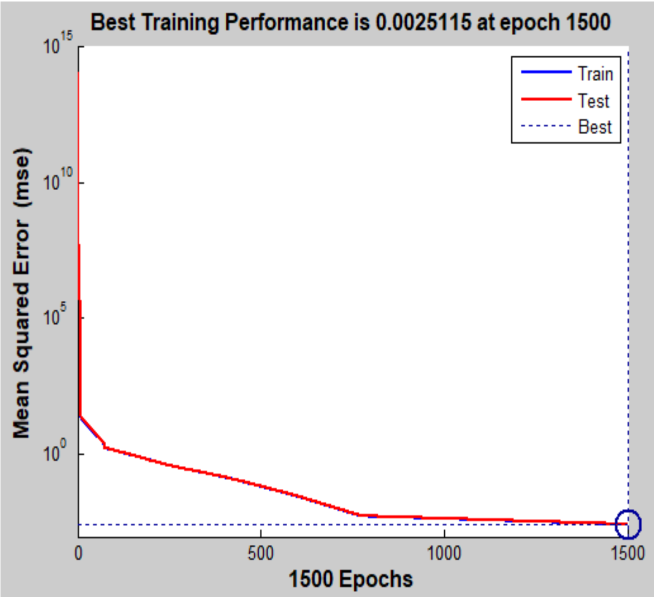

Fig. 6. BRBP's Prediction Result

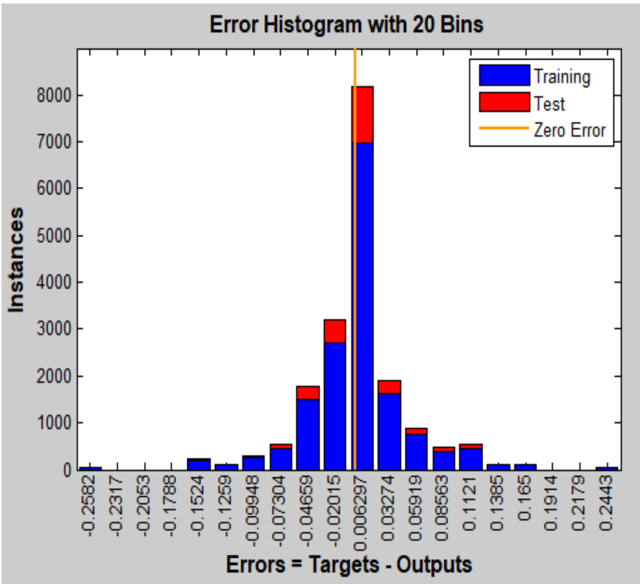

Fig. 7. BRBP's Histogram of Error Sequences

Figure 8 shows BRBP algorithm's correlation. Thus, the graphs present that the algorithm is accurate and better because the MSE is less than zero and the value of R for the training, test, and overall analysis is 1. Further, table 4 shows the performance of the enhanced LDA/GSVD.

TABLE IV
PERFORMANCE OF ANN ALGORITHM FOR LDA/GSVD USING BRBP

Dataset Sample	Mean Square Error	Regression
Training	5.6896e-05	1
Testing	6.1412e-05	1

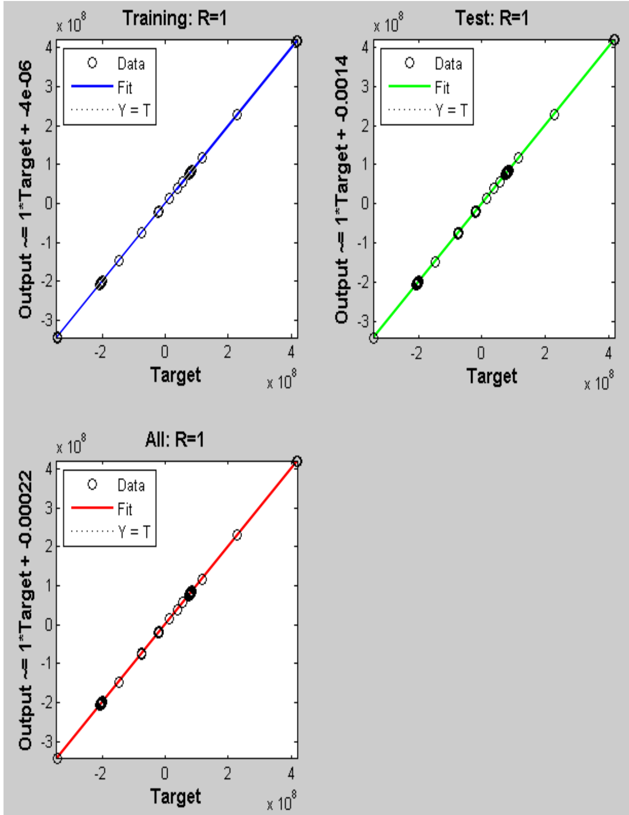

Fig. 8. BRBP's Regression Analysis

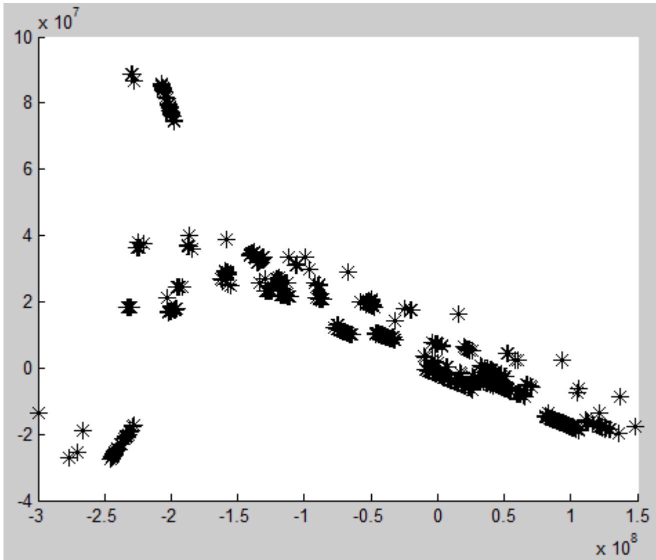

Fig. 9 Graph of the feature subspaces after applying the enhanced LDA/GSVD

It is noticeable that figure 9 above which presents the graph of the feature subspaces using the enhanced algorithm is very much similar to figure 5 which utilized the existing LDA/GSVD. It is a manifestation that the accuracy of the improved LDA/GSVD maintains the accuracy of current LDA/GSVD algorithm.

Using equation 1, Table 5 presents the computational costs of two algorithms. It is evident that the enhanced LDA/GSVD improved the computational cost by 57.14%. The values for the computational costs may be too small because there are only 1420 instances that composed the dataset.

TABLE V
COMPUTATIONAL COSTS OF THE EXISTING AND ENHANCED ALGORITHMS

Algorithm	ST[a]	ET[b]	CC[c]
Existing LDA/GSVD	08:51:02	08:51:09	7 seconds
Enhanced LDA/GSVD	09:35:43	09:35:46	3 seconds
Improvement of the Enhanced LDA/GSVD			57.14%

a. Start Time, b. End Time, c. Computational Costs

IV. CONCLUSIONS

Simulation results showed that enhanced LDA/GSVD using ANN outperformed the existing LDA/GSVD algorithm regarding computational cost during the classification of the datasets. Thus, it makes the new approach an efficient way of doing LDA/GSVD. It is also evident in the simulation that the new technique using BRBP can obtain the best performance of accuracy by increasing the number of epochs. With that, the new mechanism is highly recommended especially if the dataset has many instances and dimensions due to its lower computational cost. Moreover, implementation of the enhanced LDA/GSVD algorithm to big data will be the next research to be done.

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