Proceeding of the International Conference on Advanced Science, Engineering and Information Technology 2011

Hotel Equatorial Bangi-Putrajaya, Malaysia, 14 - 15 January 2011

ISBN 978-983-42366-4-9

A Microscopic Description of the Beta Decay of Dy^{168} using the Gamow-Teller Force

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***Abstract*— A description of the beta decay of the nucleus Dy^{168} is presented using the Gamow-Teller force in the frame of the proton-neutron Quasiparticle Random Phase Approximation (pn-QRPA). The single-particle ground states are obtained self-consistently using the Fayans energy density functional in an HFB scheme, and the quasiparticle states are built on the single-particle ground states using the BCS approximation. The calculation gives two Gamow-Teller transitions as expected, a half-life about one third of the experimental value, and a decay energy close to the experimental value.**

***Keywords*— beta decay, Gamow-Teller force, proton-neutron Quasiparticle Random Phase Approximation.**

I. INTRODUCTION

Since its invention many years ago [1], the proton-neutron Quasiparticle Random Phase Approximation (pn-QRPA) has remained the only microscopic approach available for beta decay calculation in heavy nuclei; however, it suffers from poor accuracy. Thus, the extensive work by Staudt et al. [2] using the so-called second-generation microscopic method has reproduced half-lives for about 96 % of all known neutron-rich nuclei with half-lives ≤ 60 s within a factor of 10, with an average half-life ratio y=1.67; however, their use of a simple separable Gamow-Teller force is far from satisfactory from the theoretical point of view. Meanwhile the Hartree-Fock-Bogoliubov (HFB) method is known until now as the most general apporach for the calculation of the nuclear ground state [3], and the first self-consistent ground state calculation of this type on deformed nuclei, using the so-called Fayans energy density functional ([4],[5]) has been performed by Kroemer et al. [6]. The self-consistent nuclear ground states obtained by the HFB method using this functional have been applied by Borzov et al. [7] as a basis for the calculation of beta decay half-lives in spherical nuclei using the self-consistent BCS+pn-QRPA method.

This paper uses a similar approach as in [7], however the present model employs a different interaction to obtain the excited daughter state, and the subnucleonic excitation, which was taken into account in Ref. [7], is neglected since we deal with low energies in the order of 1 MeV. The present model has been developed for the calculation of beta decay of the Gamow-Teller mode in even-even nuclei. It is interesting to apply the use of a Gamow-Teller exciting force to the description of more complex beta decay containing not only Gamow-Teller transitions, and the β^- decay from Dy^{168} to Ho^{168} meets the criteria since it contains two Gamow-Teller transition plus a $0^+ \rightarrow 1^-$ transition with an overall decay energy of 1.4 MeV, as can be seen from Figure 1 [8].

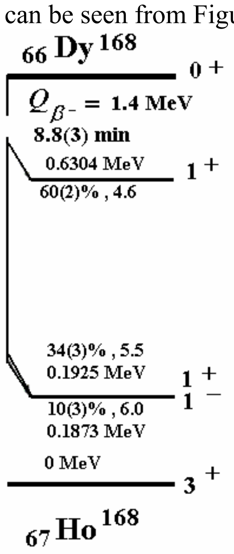

Fig. 1. Experimental β^- decay scheme in Dy^{168}. The figures next to the intensities are the $\log ft$ values.

II. THEORY

The Fayans energy density functional may be expressed as a functional of the particle normal density ρ and the anomalous density (ν, ν^{+}),

$$E[\rho, \nu, \nu^{+}] = \int d^3r \{\varepsilon_{\text{kin}} + \varepsilon_{\text{vol}} + \varepsilon_{\text{sur}} + \varepsilon_{\text{coul}} + \varepsilon_{\text{so}} + \varepsilon_{\text{pair}}\}, \quad (1)$$

where the terms in the integral are the kinetic, volume, surface, Coulomb, spin-orbit and pairing energy densities, respectively. The sum of all energy density terms except ε_{kin} is called the interaction energy density. The Fayans density functional contains Fermi parameters such as the particle density in symmetrical nuclear matter at equilibrium, $2\rho_0$, the Fermi momentum, p_F^0, the Fermi energy, ε_F^0, the sum and the difference between the proton and neutron relative densities

$$x_{\pm} = \frac{\rho^{\text{n}} \pm \rho^{\text{p}}}{2\rho_0}, \quad (2)$$

as well as their functions and derivatives,

$$f_{\pm} = \frac{1 - h_{1\pm}x_{\pm}}{1 + h_{2\pm}x_{\pm}}, \qquad f' = \frac{d}{dx_{+}} f(x_{+}), \quad (3)$$

where $h_{1\pm}$ and $h_{2\pm}$ are parameters of the functional. The single-particle potential and the strength of the two-nucleon p-h interaction may be derived from the derivative of the interaction energy density with respect to particle density ρ, while the particle-particle (p-p) interaction may be derived from the derivative of the pairing energy with respect to pairing (anomalous) densities (ν, ν^{+}).

In HFB theory one looks for the most general product wave functions consisting of independently moving quasi-particles. Within this approximation, the Hamiltonian reduces to two average potentials, the self-consistent field Γ, which is already known from the Hartree-Fock theory, with matrix elements

$$\Gamma_{ll'} = \sum_{qq'} \bar{v}_{lq'l'q} \rho_{qq'}, \quad (4)$$

and an additional pairing field, Δ known from the BCS theory with matrix elements

$$\Delta_{ll'} = \frac{1}{2} \sum_{qq'} \bar{v}_{ll'qq'} \nu_{qq'}. \quad (5)$$

The field Γ, also called the normal pairing potential, contains all the long-range p-h correlations which eventually lead to a deformed ground state, whereas the field Δ, also called the anomalous pairing (tensor) pairing potential, sums up the short-range pairing correlations that can lead to a phase transition and a superfluid state. The BCS quasi-particles is a special type of quasi-particle defined by a special Bogoliubov transformation. Even though Γ (and $h = t + \Gamma$, with t kinetic energy) is not diagonal, it is convenient to define single-particle energies by

$$\varepsilon_k = h_{k\bar{k}}. \quad (6)$$

Analogously one may define single-particle energy gap parameters by

$$\Delta_k = \Delta_{k\bar{k}}. \quad (7)$$

It is convenient to define an average pairing gap in the neighborhood of the Fermi level by averaging the values of Δ_k directly above and below the Fermi level,

$$\Delta_F = \frac{\Delta_{F+1} + \Delta_{F-1}}{2}. \quad (8)$$

The HFB ground state eigenfunctions are obtained by solving the HFB equations

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}, \quad (9)$$

where $h = \varepsilon + \Gamma - \lambda$, ε and λ are respectively the single-particle energy and the chemical potential (equal to the Fermi energy ε_F). The E_k are now the HFB quasiparticle energies. The (sub)matrices **U** and **V** determine uniquely the HFB quasiparticle operator. In the present work, the iterative procedure to obtain a self-consistent solution to the HFB equation (9) starts with the Saxon-Woods single-particle potential [9] as the initial approximation to $V^{\text{n/p}}(\vec{r})$, and is described in detail in Ref. [6].

The selection rules for a Gamow-Teller transition are that the magnitude of the change in spin and isospin must equal 0 or 1 (not 0 → 0), and that the nuclear parity must be conserved. The matrix elements of the nuclear Gamow-Teller transitions for a β^- decay from an initial nuclear state $|N_i\rangle$ to the final state $|N_f\rangle$ may be expressed as

$$M_{\text{fi}}^{-} = \langle N_f | \sum_{j=1}^{A} \vec{\sigma}(j) \vec{\tau}^{-}(j) | N_i \rangle. \quad (10)$$

Using the spherical coordinate representation, the Gamow-Teller matrix elements may be decomposed into some geometric factor and a reduced matrix [10]

$$M_{\text{fi}}^{-} = -c_A (J_i M_{i1} \mu | J_f M_f) \frac{\langle N_f | \sum_{j=1}^{A} \vec{\sigma}(j) \vec{\tau}^{-}(j) | N_i \rangle}{\sqrt{2J_i + 1}}, \quad (11)$$

where the axial vector renormalization constant $c_{\text{A}} = 1.26$ [11], and the parent nuclear spin $J_{\text{i}} = 0$ in this investigation. The reduced transition probability B_{GT} is defined as

$$B_{\text{GT}} = \frac{|M_{\text{fi}}^{-}|^2}{2J_{\text{i}} + 1}. \quad (12)$$

The total half-life $T_{1/2}$ may be obtained by summing over all energetically allowed transitions

$$\frac{1}{T_{1/2}} = \sum_n \frac{1}{t_n} = \frac{c_{\text{A}}^2}{D} \sum_n f_n B_{\text{GT}}(n), \quad (13)$$

where $D = 6146.6 \pm 7$ s [12].

In the pn-QRPA model, the excitation from the parent nucleus into the daughter nucleus is mediated through the creation of pn-QRPA phonons [13]

$$A^{+}_{\varpi} = \sum_{\text{pn}} \left(X^{\text{pn}}_{\omega\mu} \alpha^{+}_{\text{p}} \alpha^{+}_{\bar{\text{n}}} - Y^{\text{pn}}_{\omega\mu} \alpha_{\text{p}} \alpha_{\bar{\text{n}}} \right), \quad (14)$$

where X and Y are called forward (p-h) and backward (h-p) amplitudes, respectively, with ω being the phonon energy and μ the phonon multipolarity, which in the Gamow-Teller case is equal to 1^+. In a self-consistent HFB+pn-QRPA model, the QRPA ground state should be approximated by the HFB ground state; in the present work, however, the self-consistent HFB single-particle ground states are used to calculate the simpler BCS quasi-particle states, and in this BCS+pn-QRPA model the QRPA matrix equation of the following form is obtained

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X_\nu \\ -Y_\nu \end{pmatrix} , \qquad (15)$$

where ω_ν is the ν-th eigenvalue of the excitation energy, and the matrix elements of the matrices *A* and *B* depend on the p-h force obtained from the Migdal-Larkin force [14]

$$F_{\text{p-h}} = C_0 \delta(\vec{r}_1 - \vec{r}_2)\{f + f' \vec{\tau}_1 \cdot \vec{\tau}_2 + g\vec{\sigma}_1 \cdot \vec{\sigma}_2 + g' \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2\}, \qquad (16)$$

where g' is the Gamow-Teller parameter, and C_0 a force constant. The parameters *f* and *f'* are density-dependent, while *g* and *g'* are density-independent parameters. The value $\rho_0 = 0.17/2$ has been used in this work. In eq. (13), f_n are the corrected Fermi integrals for vector transition in the *n*th nuclear Gamow-Teller transition, which may be obtained from the value of log f_A [15] and the relativistic correction factor [16].

From the solutions of the QRPA matrix equation (15), excitation energy eigenvalues ω(*i*) and Gamow-Teller transition amplitudes are obtained. The decay energy for each transition, $Q_\beta(i)$ is calculated using

$$Q_\beta(i) = \lambda_{\text{n}} - \lambda_{\text{p}} + \Delta m_{\text{nH}} - \omega(i) , \qquad (17)$$

where $\Delta m_{\text{nH}} = 0.782$ MeV, and $\lambda_{\text{n}} - \lambda_{\text{p}}$ is the difference of chemical potentials of neutrons and protons.

Except for the determination of the proton-proton and the neutron-neutron p-p pairing force parameters, the numerical procedure for the ground state is identical with that in Ref. [6], using the so-called DF3 Fayans energy density functional parameter set. The proton-proton (f_p) and the neutron-neutron (f_n) p-p pairing force parameters appearing in the ground state calculation are chosen as to produce the average pairing gap near the Fermi level (eq. 8) equal to the Lipkin-Nogami gap parameter ([17]-[19]), $f_p = -0.727$, $f_n = -0.675$. To handle the large matrices involved in the QRPA calculation, one is forced to perform truncation of the configuration space. The matrix element selection has been carried out carefully in the present work in order to reproduce the theoretical Ikeda Gamow-Teller sum rule [20] as close as possible. Using $\hbar\omega_0 = 41A^{-1/3}$, the selection of the configuration space was performed using the following rules. (a) The first step is, all matrix elements whose pair components have single-particle energies within are included, and (b) all of the remaining matrix elements are ordered according to their magnitudes, and only the largest matrix elements are included so the total number of included matrix elements reach 900.

III. Results and Discussion

The reference Lipkin-Nogami gap parameters in Dy^{168} for proton and neutron are 1.05 and 0.84, respectively, and these are reproduced exactly in the ground state calculation. The calculated binding energy is 1364 MeV, reproducing well the experimental value of 1363 MeV [21], giving a deviation of 0.07 per cent. Figure 2 shows the calculated and experimental reduced transition probabilities, B_{GT} , as a function of excitation energy in daughter nucleus relative to the 1^+ state. Our result reproduces exactly two Gamow-Teller transitions as expected having the correct relative magnitudes of the reduced transition probabilities. The calculated energy difference of both Gamow-Teller transisions and their reduced transition probabilities are smaller by a factor of about two compared to experimental data. As shown in Table 1, the calculated total half-life is about one third of the experimental value, which is not satisfactory, while the calculated decay energy of 1.63 MeV is close to the experimental 1.4 MeV.

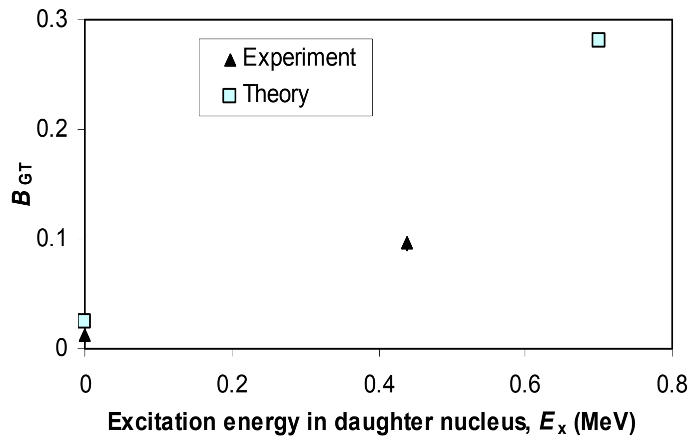

Fig. 2. The calculated and experimental reduced transition probabilities, B_{GT} as a function of excitation energy of daughter nucleus.

Table 1. Calculated half-life and decay energy compared with experimental values.

Half-life (min)		Decay energy (MeV)	
Calc.	Exp.	Calc.	Exp.
3.01	8.8(3)	1.63	1.4

IV. Conclusions

In summary, the complex β^- decay in the nucleus Dy^{168} has been described using the proton-neutron Quasiparticle Random Phase Approximation (pn-QRPA) on a BCS quasi-particle basis, taking into account the Gamow-Teller transitions only. In view of the rather crude approximation made and the lack of accuracy in most previous microscopic beta decay calculations, the results are encouraging and motivate more extensive calculations to include other complex nuclear beta decays.

REFERENCES

[1] J.A. Halbleib, R.A. Sorensen, "Gamow-Teller beta decay in heavy spherical nuclei and the unlike particle-hole RPA", *Nucl. Phys. A.* vol. 98, p. 542, 1967.

[2] A. Staudt, E. Bender, K. Muto, and H.V. Klapdor-Kleingrothaus, "Second-generation microscopic predictions of beta-decay half-lives of neutron-rich nuclei", *At. Data and Nucl. Data Tables* vol. 44, p. 79, 1990.

[3] A.L. Goodman, "Hartree-Fock-Bogoliubov Theory with Applications to Nuclei ", *Adv. Nucl. Phys.* vol. 11 , p. 263, 1979.

[4] S.A. Fayans, ``Towards a universal nuclear density functional", *JETP Lett.* vol. 68 no. 3, p. 169, 1998.

[5] S.A. Fayans, S.V. Tolokonnikov, E.L. Trykov, and D. Zawischa, "Nuclear isotope shifts within the local energy-density functional approach", *Nucl. Phys. A* vol. 676, p. 49, 2000.

[6] E. Kroemer, S.V. Tolokonnikov, S.A. Fayans, and D.Zawischa, "Energy-density functional approach for non-spherical nuclei", *Phys. Lett. B* vol. 363, pp. 12-16, 1995.

[7] I.N. Borzov, S.A. Fayans, E. Kroemer, and D. Zawischa, "Microscopic calculations of β -decay characteristics near the $A=130$ r-process", *Z. Phys.A* vol. 355, p. 117, 1996.

[8] R. M. Chasteler, J. M. Nitschke, R. B. Firestone, K. S. Vierinen, and P. A. Wilmarth, "Decay studies of the neutron-rich isotopes ^{168}Dy and ^{168}Ho^{g} and the identification of the new isomer ^{168}Ho^{m} ", *Phys. Rev. C*, vol. 42 no. 5, p. R1796, 1990.

[9] D. Zawischa, and J. Speth, "Orbital and spin-flip magnetic dipole resonances in heavy nonspherical nuclei", *Nucl. Phys. A* vol. 569, pp. 343c-352c, 1994.

[10] K. Grotz, and H. V. Klapdor, "The Weak Interaction in Nuclear, Particle and Astrophysics", Bristol, UK: IOP, 1990.

[11] Particle Data Group (M. Aguilar-Benitez *et al.*), 1994, "Review of Particle Properties", *Phys. Rev.D* vol. 50, p. 1173.

[12] J.C. Hardy, and I.S. Towner, "Superallowed beta decay of nuclei with $A \geq 62$: the limiting effect of weak Gamow-Teller branches", *Phys. Rev. Lett.* vol. 88, pp. 252501-252504, 2002.

[13] P. Ring, and P. Schuck, "Nuclear Many-Body Problem", Berlin, Germany: Springer, 2000.

[14] A.B. Migdal, and A.I. Larkin, *Nucl. Phys.* vol. 51, p. 561, 1964.

[15] N.B. Gove, and M.J. Martin, "Log-*f* tables for beta decay", *Nucl. Data Tables* vol. 10, pp. 205-317, 1971.

[16] D.H. Wilkinson, and B.E.F. Macefield, "A parametrization of the phase space factor for allowed β-decay", *Nucl. Phys. A* vol. 232, p. 58, 1974.

[17] R.B. Firestone, S.Y.F. Chu, and C.M. Baglin, "Table of Isotopes: 1998 Update 8th edition", Wiley Interscience, 1998.

[18] P. Moeller, and J.R. Nix, "Nuclear Pairing Models", *Nucl. Phys. A* vol. 536, pp. 20-60, 1992.

[19] P. Moeller, J.R. Nix, and K.-L. Kratz, "Nuclear Properties for Astrophysical Applications and Radioactive-Ion-Beam", *At. Data Nucl. Data Tables* vol. 66, p. 131, 1997.

[20] M.C. Vetterli, O. Häusser, R. Abegg, W.P. Alford, A. Celler, D. Frekers, R. Helmer, R. Henderson, K.H. Hicks, K.P. Jackson, R.G. Jeppesen, C.A. Miller, K. Raywood, S. Yen, "Gamow-Teller strenGamow-Tellerh deduced from charge exchange reactions on Fe^{54} at 300 MeV", *Phys. Rev.* vol. 40, p. 559, 1989.

[21] G. Audi, A. H. Wapstra, and C. Thibault, *Nucl. Phys. A* vol. 729, pp. 337-676, 2003.