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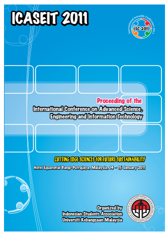

Maple Toolbox for Switched Stabilizing Controller

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Abstract— **This paper is celebrating the increment of interest in the application of computer algebra in control system analysis. A Maple toolbox for stabilizing state feedback controllers for a class of switched system is presented. The attention is focused on finding the existence of common Lyapunov function (CLFs), as this ensures stability for arbitrary switching sequences between several subsystems. The system considered here are restricted to second order linear systems. In order to find the common Lyapunov function and the ability of the Maple software, the toolbox is proved to be less computational demanding compared to a lot of methods that has been solved by Linear Matrix Inequalities (LMI).**

Keywords— **Lyapunov function, stability, switched system**

I. INTRODUCTION

Switched system is a class of known practical implication in hybrid system. This currently receiving a deep attention by the control community is actually made up from a collection of linear subsystems with rules that govern switching between these subsystems [1].

There are many researches have been done on the subject of switching systems. Among the researches, the most popular topics are the stability of the switched system using various approaches. Within the most popular topics of stability of switched systems, Linear Matrix Inequalities (LMI) is the favourite method to solve the problems. [2] formulated using LMI in stability analysis on the regulation of the angle of attack on aircraft and PI control of automatic transmission vehicle and manual transmission by [3].

In the meanwhile, [4] proved that by using LMI approach, all eigenvalues of each linear subsystem must be inside the circle of left-hand half of complex plane in order to stabilize the switched system.

Computer algebra is the best medium to solve LMI problems. In [5], they presented the LMI Control Toolbox for use with Matlab. This toolbox offers numerical algorithms for solving generic LMI problems for LMI-based control design. The researches whose develop the algorithm especially for stabilize the switched controller may need this toolbox to solve their problem and furthermore to proof the stability of the system.

[6] used the Matlab to design a toolbox which demonstrated the stabilizing controller for a class of switched system. Matlab become the command computational algebra software to solve such numerical algorithm. This Matlab toolbox is designed as the complement to the research of finding the algorithm for stabilizability of a class of switched systems in [7]. This algorithm is said to be less computational demanding. This toolbox presented the controller coefficient for each subsystem which ensures the stability of the whole system.

However, the Maple will be used to strengthen the findings in [7]. Maple is the essential technical computing software using symbolic computational for complex mathematical calculations and designed for control engineering with shortest language. Maple toolbox is the interactive toolbox for stabilizing controller for a class of switched system. This interactive toolbox allows the user to understand the process to stabilize each subsystem before stabilize the whole system.

The class of switching system considered here is denoted by

$$\dot{x} = A_i x + B_i u \qquad (1)$$

where $x \in R^2, u \in R, A \in R^{2x2}, B \in R^{2x1}$ and $i = 1, \dots, N$. Our attention is to find the existence of common Lyapunov function (CLFs) which ensure quadratic stability for arbitrary switching sequences of system (1).

Definition 1 System (1) is said to be quadratically stabilizable if there exists a set feedback control laws $u_i = A_i + B_i K_i$ such that $A_i + B_i K_i, i = 1,2, \ldots N,$ share a common quadratic Lyapunov function $x^T M x$.

[8] and [9] obtain the solution to the problem posed by Definition 1 with the following LMI:
If there exist a scalar γ>0 and a positive definite matrix P, such that

$$A_i P + P A_i^T - \gamma P^T B_i B_i^T P < 0, \qquad \forall i \in \mathfrak{J} \tag{2}$$

where

$$K_i = (\tfrac{\gamma}{2}) B_i^T P.$$

Later after that, [4] provided the sufficient condition for a closed loop switched system to be asymptotically stable with the following LMI:

If there exist a symmetric positive definite matrix $W \in \mathbb{R}^{n \times n}$ and matrices $Z_j \in \mathbb{R}^{m_j \times n}, j = 1, \ldots, N$, such that

$$A_j' W + W A_j + B_{2j} Z_j + Z_j' B_{2j}' < 0, \;\; j = 1, \ldots, N \tag{3}$$

then

$$K_j = Z_j W^{-1}, \;\; j = 1, \ldots, N \tag{4}$$

These methods allows the determination of a switched control law that guarantee the assignment of the poles of each linear subsystems of the switched system inside an arbitrary circle in the open left-half complex plane. However, the computational burden while solving the problems using most of computer algebra such as Matlab, Scilab and Maple would be a limiting factor.

II. THE METHOD

A complete description with proofs of the method can be found in [6]. The following pseudo code implements this method algorithmically using symbolic computation software Maple 13.

```
> with(LinearAlgebra) :
A[1] := A1 : A[2] := A2 : A[3] := A3 : A[4] := A4 : A[5] := A5 :
    A[6] := A6 : A[7] := A7 : A[8] := A8 : A[9] := A9 : A[10]
    := A10 :
B[1] := B1 : B[2] := B2 : B[3] := B3 : B[4] := B4 : B[5] := B5 :
    B[6] := B6 : B[7] := B7 : B[8] := B8 : B[9] := B9 : B[10]
    := B10 :
for n from 1 to k do AB[n] := A[n] · B[n];end do:
for n from 1 to k do ABi[n] := AB[n]–((((⟨B[n]|AB[n]⟩))^-1 · (A[n]
    · AB[n]))[2] · B[n]);end do:
for n from 1 to k do C[n] := (⟨ABi[n]|B[n]⟩)^-1;end do:
for n from 1 to k − 1 do T[n] := C[1] · C[n + 1]^-1;end do:
for n from 1 to k − 1 do PSz[n] := T[n][1, 2]*T[n][2, 1]
    + T[n][1, 1]*T[n][2, 2];end do:
for n from 1 to k − 1 do QSz[n] := T[n][1, 1]*T[n][1, 2]; end do:
for n from 1 to k − 1 do P[n] := T[n][1, 1] / T[n][2, 1]; end do:
for n from 1 to k − 1 do Q[n] := T[n][1, 2] / T[n][2, 2]; end do:
R := [seq(solve((P[n] + Q[n] − P[n + 1] − Q[n + 1])x + (P[n]
    *Q[n]) − (P[n + 1]*Q[n + 1])), n = 1 ..k − 2)] :
vx := (Rhigh − Rlow)/2 + Rlow :
ve := (ehigh − elow)/2 + elow :
M[1] := [ 1    vx        ]
        [ vx   vx^2 + ve ] :
for n from 1 to k − 1 do  M[n + 1] := T[n]^%T · (M[1] · T[n]);end
    do:
for n from 1 to k − 1 do  M[n + 1] := T[n]^%T · (M[1] · T[n]);end
    do:
```

III. THE TOOLBOX

Maple toolbox is designed to determine the stabilizability of a class of switched system. This toolbox is made in Maple worksheet, so the input toolbox and the output are included in the same page. It is divided into two sections which is the input section and the output section. The user only required three simple steps in order to find the stabilizability of their system.

Step 1: Choose the number of switching models
When the user press on the sentence of step one, the window will bring the user to "Number of switching Models" in the same page of the window (fig. 1). From the combo box, the user may choose the required number of switching subsystem needed. Besides the combo box, the square box will show the number of subsystems chosen for confirmation. This toolbox offers to stabilize until ten subsystems.

Step 2: Key in the value of states for every models selected
This step necessitates the user to key in the state of every subsystem.

Step 3: Press the !!! button at the top of the window to run the toolbox
By pressing the !!! button, Maple will start execute the entire worksheet according to our command.

From step 3, Maple toolbox is ready to show not only the stabilizability of the system, but this toolbox also let the user to see every step until the confirmation of the stabilizability of system.

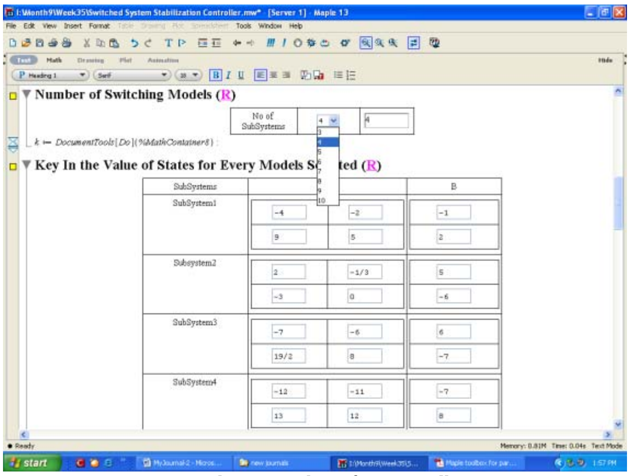

Fig. 1 The screenshot of input toolbox

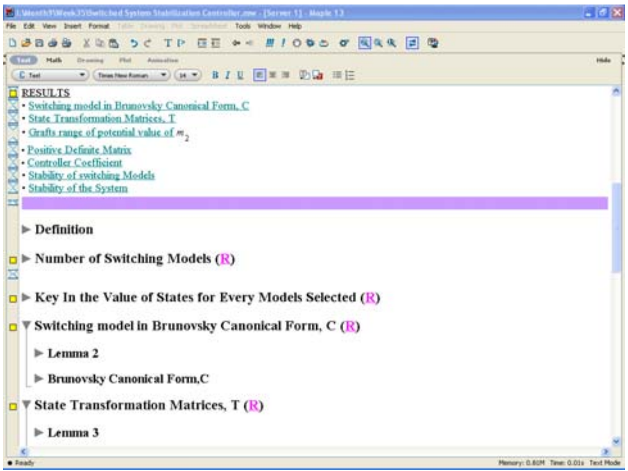

Fig. 2 The screenshot of output toolbox

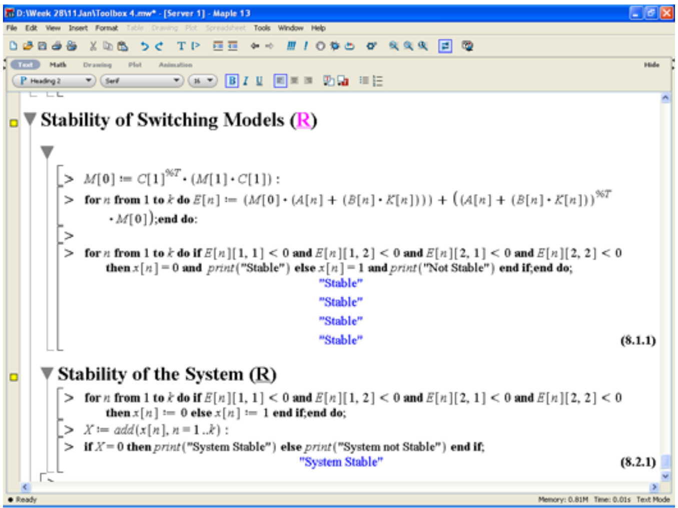

Fig. 3 The screenshot of stability of the system

IV. EXAMPLE

Picked from [7], consider a case of a switching system as (1) which has 4 subsystems as below

$$A_1 = \begin{bmatrix} -4 & -2 \\ 9 & 5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & -\frac{1}{3} \\ -3 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -7 & -6 \\ \frac{19}{2} & 8 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -12 & -11 \\ 13 & 12 \end{bmatrix}, \quad B_4 = \begin{bmatrix} -7 \\ 8 \end{bmatrix}$$

Key in all the values of A_i and B_i where $i = 4$ as in fig. 1 before execute the toolbox. After less than 2 seconds with at the average of 42 megabytes memories in the computer, the stability of switched system is identified.

In the meanwhile, fig. 2 visualizes the output section of the toolbox which provides seven important values involved. Below the purple line in fig. 2 explain the complete lemma, algorithm and theorem that already been proofed in [7] for the stabilizability of the system. For example, the first step in determining the stabilizability of the system, each subsystem is converted into Brunovsky controllable canonical form. So, this toolbox introduce the first lemma which clarify the conversion of subsystem into Brunovsky canonical form before implement the lemma in Maple command and solve the problem of this section. At the end of Maple Toolbox, the window displays the stability of every subsystem and the final finding which is the stability of the whole system (fig. 3).

V. CONCLUSION

A Maple toolbox for stabilizing state feedback controller for a class of switched system has been developed. The toolbox expressed less computationally demanding compared to a lot of method that using LMI. The usage of symbolic computational might reduce the error of final result compare to most of the software using numerical computational in solving the problems. The finding is guaranteed the stability for arbitrary switching sequences between several subsystems as the existence of common Lyapunov function (CLFs). The toolbox also becomes very useful for the beginners in research of control engineering as they able to study step by step in determining the stabilizability of switched system.

In the future, the toolbox will be expanding by analyzing the stabilizability of switched system including the Step Response and Pole/Zero Location before and after stabilizing the switched system.

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