Optimization of Circular Side Door Beam for Crashworthiness Analysis

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Abstract— Structural optimization related to crashworthiness and energy absorption capability is particularly important to the automotive industry. The optimization involves highly nonlinear computational analysis and design with many material and structure parameters. This paper presents a crashworthiness design of the circular side door beam which attach to the side door structures. The response surface method (RSM) is utilized to formulate the complex crashworthiness design problem in the case of optimization. In this study, side door beam will be optimized. The beams in circular shape were studied and compared. The focus is on finding an optimum cross-section shape of the beam in order to improve the energy absorption character. An optimization problem is formulated to find the maximum energy absorbed with the maximum peak load as a constraint, the shape of the beam cross section and the thickness as variable. The structure optimum design and simulation analysis of automobile side-door beam was carried out by using Finite Element (FE) method.

Keywords— Side door beam, crashworthiness, optimization, RSM.

I. INTRODUCTION

Crashworthiness design of vehicles for passive safety aims to obtain a strong, crushproof passenger surviving cell connected to several components able to absorb and dissipate energy in a stable and controlled manner. In the case of side impact collision, side door beam which is installed in the door played a very important role to absorb a lot of energy, thereby reducing the amount of energy transferred to the occupants. Investigation on crushing energy absorption, induced decelerations and consequent applied forces of this kind of structures has been carried out by many researchers. Several theoretical models have been developed and refined to predict the absorbed energy of side door beam [1], [2]. Numerical analyses and experimental validation have been carried out on the side structure and a door trim [3], [4]. However, few attempts have been made to optimize the crashworthiness of side door structure by applying structural optimization techniques [5], [6], [7].

Most research [3] – [7] related to the side door beam has been performed with the door assembly or the BIW (body-in-white) system on account of test regulations. That is, it is not easy to predict the effectiveness of the side door beam during the design process. In this research, the performance of a side door beam is predicted using only the door beam and not the BIW system. In considering the effect of the BIW system, an equivalent modelling is utilized. The optimization using response surface method (RSM) is used as a design approach to determine the thickness and the cross sectional shape of a side door beam. The objective is to maximize the energy absorption of a side door beam with respect to the mass density. The design variables are the thickness and the shape cross section.

The model of the side-door impact beam impacting with the rigid pole was set up. The numerical simulations were carried out using the software LS-DYNA. Attention was focused upon finding an optimum cross-section shape of the beam in order to improve the energy absorption character.
II. PROBLEM MODELING

A thin-walled beam with the optimized cross-sectional dimensions should absorb as much crash energy as possible per unit weight, as explain by Hou [8]. This means, the specific energy absorption (SEA) should be set as an objective function and maximize during the optimum design. The SEA is defined as

\[
\text{SEA} = \frac{\text{Total energy absorb, } E_{\text{total}}}{\text{total structural weight}}. \quad (1)
\]

Two factors have to be considered in assigning constrains on this problem. The first one is human safety issues which is the maximum crushing force that occurs during the crash should be under certain criteria. It is very important in the automotive design and manufacturing. The other is the cross-section of thin-walled beam; the two design variables which are cross-section, \( a \) and the wall thickness, \( t \) (figure 1) should take values between their upper and lower bounds.

![Typical Circular Side Door Beam Cross-section](image)

In this study, only the beam cross-sections and the wall thickness has been consider in optimization. Thus, this optimization problems is modeled as

Maximize : \( \text{SEA} (a, t) \)

Constraints : \( P_m \leq \text{Criteria} \)
\( a^L \leq a \leq a^U \) and \( t^L \leq t \leq t^U \),

\[
(2)
\]

Where \( a^L, a^U, t^L, t^U \) are the lower and upper bounds of the design variables \( a \) and \( t \), respectively.

III. RESPONSE SURFACE METHOD (RSM) [15]

A surrogate model technique represented by the Response Surface Method (RSM) is one of the prevalent techniques to model highly-nonlinear systems beside a Genetic Algorithm (GA). As a non-gradient global method, GA demonstrates it capability of dealing with crashworthiness design problems. However, a problem of GA is relatively low computing efficiency. The ideas of surrogate model is to employ simple basic functions, typically the polynomial to approximate complex crashing response of a structure, which avoids the FE driven sensitivity analysis. In this context, Fang et al. [8] compare difference basic functions on a fitting accuracy of the response functions. They suggested that the polynomial basic function generally provides a good approximation to model the energy absorption.

In this study, the response of the thin-walled circular side door beam is the \( \text{SEA} (a, t) \), which is approximate using the series of the basic function in a form of

\[
\hat{y}(x) = \text{SEA}(a, t) = \sum_{i=1}^{n} \beta_i \phi_i(a, t), \quad (3)
\]

Where \( n \) represents the number of basic functions \( \Phi(a, t) \). In this paper, a polynomial is used to build up this basic function to formulate the SEA. In Eq. 3, \( \beta_i \), known as a regression coefficient, are estimated using the method of least squares. Suppose we have \( m(m>n) \) observations (obtains from FEA) for the yielded response \( y_i (y_1 - y_m) \) based on the \( m \) sampling design points \( (a, t)_i \), the least squares function is therefore expressed as

\[
L = \sum_{i=1}^{m} \varepsilon_i^2 = \sum_{i=1}^{m} \left[ y_i - \sum_{j=1}^{n} \beta_j \phi_j(a, t) \right]^2. \quad (4)
\]

Where the design points \( (a, t)_i \) are selected from the specified design space, \( \varepsilon_i \) is the error between the response \( y_i \) observed at these points. Afterward, the coefficient vector \( b = (\beta_1, \beta_2, \ldots, \beta_n) \) can be determine by \( \delta L/\delta b = 0 \), which is,

\[
b = (\Phi^T\Phi)^{-1}\Phi^Ty, \quad (5)
\]

Where \( \Phi \) denotes the matrix consisting of basic functions evaluated using \( m \) sampling points, which is

\[
\Phi = \begin{bmatrix}
\phi_1(a, t) & \cdots & \phi_n(a, t)
\vdots & \ddots & \vdots
\phi_1(a, t)_m & \cdots & \phi_n(a, t)_m
\end{bmatrix} \quad (6)
\]

By substituting Eq. (5) into Eq. (3), the response surface (RS) model is created and the response function \( \text{SEA} (a, t) \) can be fully determined. The relative error (RE) between the observed response at those sampling points \( y(x) \) and the original response \( y(x) \) is

\[
\text{RE} = \frac{[\hat{y}(x) - y(x)]}{y(x)}. \quad (7)
\]

Other two important properties in evaluating the model’s accuracy are the sum of the residuals (SSR) and the total sum of squares (SST), which are

\[
\text{SS}_E = \sum_{i=1}^{m} (y_i - \hat{y}_i)^2, \quad (8)
\]

\[
\text{SS}_T = \sum_{i=1}^{m} (y_i - \bar{y}_i)^2, \quad (9)
\]

Where \( y_i \) is the mean value of \( y_i \).
The model fitness can be evaluated based on the F statistic, coefficient of multiple determinations $R^2$, adjusted $R^2$ statistic and root means square error (RMSE) respectively, which are calculated as

\[
F = \frac{(SS_T - SS_E)/n}{SS_E/(m - n - 1)}
\]

(10)

\[
R^2 = 1 - \frac{SS_E}{SS_T}
\]

(11)

\[
R_{adj}^2 = 1 - \frac{m - 1}{m - n} (1 - R^2)
\]

(12)

\[
RMSE = \sqrt{\frac{SS_E}{m - n - 1}}
\]

(13)

According to the RSM theory, the larger value of $R^2$ and $R_{adj}^2$, and the smaller value or RMSE indicate the better fitness of RS model.

IV. PROBLEM MODELING

A. FE Model of Circular Side Door Beam

A beam structure considered in this study is circular shape with the hollow cut. The structure based on the Proton Wira which is manufacture in Malaysia as shown in figure 2. The side length $a$ of the cross sections and the thickness $t$ of the thin wall are chosen as design variables. The column length of $L = 900$mm remains as a constant in all design cases. The beam structures impact onto a pole with an initial velocity of $15\text{m/s}$.

![Side door structure](image)

**Fig. 2 Side door structure**

Figure 3 shows the schematic of the computational model. The structures are made of the aluminum alloy AA6061-T4 with the material properties of density, $\rho = 2.7 \times \text{kg/m}^3$, Young’s modulus, $E = 70.0$GPa, Poisson’s ratio $\nu = 0.28$, yielding stress, $\sigma_y = 110.3$MPa and tangent modulus, $E_t = 450$MPa.

The plastic kinematic hardening material model with material type 3 in LS-DYNA is considered in the constitutive model. Since the aluminum is insensitive to the strain rate, this effect is neglected in the FE modeling [7]. The FE models, including the selection of elements, the definition of contact and the optimization process are the same for all these five different design cases except that the different cross-sectional layouts are used. Regarding the contact, nodes to surface contacts between the beam and a pole is defined, and meanwhile, a single surface contact algorithm provided by LS-DYNA3D is also utilized to consider the self-contact between the shell elements.

B. Sampling Design

In order to derive the response functions of SEA, a series of sampling points (based on $t$ and $a$) are selected in a design space to provide sampling design for FEA. The design ranges are determined according to a previous research [8] and the optimization problem is defined as

\[
\text{Maximize : } \quad \text{SEA (t,a)}
\]

\[
\text{Constraint: } P_m \leq 90 \text{kN},
\]

\[
30 \text{mm} \leq a \leq 50 \text{ mm}
\]

\[
1.0 \text{ mm} \leq t \leq 3.0 \text{ mm}
\]

(14)

In a design range of $t$ and $a$, five level full factorial design is used for sampling, which resulted in $5^2 = 25$ design points evenly distributed within their design range. The FEA results for these 25 design points with difference values of $t$ and $a$ are used. The response functions of SEA are derived from the FEA results based on the 25 beam models.

C. Response Surface Model

RS model can be developed after having all the FEA results on 25 thin-walled beam models. The polynomials are used as a basic function in generating these RS models. In this section, linear, quadratic and cubic polynomials are used in order to validate the selection of design points and the orders...
of polynomial. The accuracies were evaluated by using following Eqs. (8)- (13). The regression coefficients $\beta_i$ for these polynomial are determined using Eqs. (5) and (6). The response polynomials function of SEA are:

**Linear form**

$$SEA(a,t) = 96.6604 + 0.1136a + 2.8834t$$

**Quadratic form**

$$SEA(a,t) = -4.4457 + 6.0882a + 4.1656t - 0.0868a^2 + 0.0523at - 0.779t^2$$

**Cubic form**

$$SEA(a,t) = -61.4711 + 11.1033a + 5.7910t - 0.2272a^2 - 0.1670at + 0.3843t^2 + 0.0013a^3 + 0.0024a^2t + 0.0135t^2a - 0.2724t^3$$

The results of approximations are summarized in Table 1 for the circular side door beam. The largest value of $R^2$ and $R^2_{adj}$ and the smaller value RMSE goes to quadratic polynomial. The table clearly show that of all these three Rs model, the quadratic polynomial function consistently provide the best approximation. The data also validate the factorial selection of the evenly distributed 25 design points in the entire design domain.

<table>
<thead>
<tr>
<th>RS model</th>
<th>$R^2$</th>
<th>$R^2_{adj}$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear polynomial</td>
<td>0.2541</td>
<td>0.9322</td>
<td>7.6599</td>
</tr>
<tr>
<td>Quadratic polynomial</td>
<td>0.9616</td>
<td>0.9899</td>
<td>0.4531</td>
</tr>
<tr>
<td>Cubic polynomial</td>
<td>0.9497</td>
<td>0.9698</td>
<td>0.7420</td>
</tr>
</tbody>
</table>

As a result of least square procedure, the quadratic response functions of maximum peak load, $P_m$ is respectively given as

$$P_m(a,t) = 27.6594 - 0.8730a - 9.9197t + 0.0163a^2 + 1.05769at + 2.6343t^2$$

Figure 4 shows the RSM result of quadratic polynomial for circular side door beam. The results of SEA increased with the increasing in the beam thickness. Besides that, the increments of SEA result in contra with the beam diameter.

Afterwards, the optimal design is obtained using constrained nonlinear multivariable optimization algorithm, which is $a=30\text{mm}$, $t=2.34\text{mm}$ with SEA of 109.25 kJ/kg and $P_m$ of 90.00kN.

**V. CONCLUSIONS**

This paper presents the crashworthiness design of the circular side door beam which is attach to the side door structures. Response surface methods are used to optimize beam cross-section with respect to the maximum specific energy absorption and the maximum peak load as a constraint. Three different type of polynomials function of response surface model were used. The error evaluation shows that the quadratic polynomial is the best function in this study. By using an optimization algorithm, the optimum circular side door beam cross-sectional is 30 mm in diameter, and 2.34mm as a thickness.

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