Robust Approach of Optimal Control for DC Motor in Robotic Arm System using Matlab Environment

Hasan Abbas Hussein Al-khazari\textsuperscript{a,b,1}, Mohammed Abdulla Abdulsada\textsuperscript{a,2}, Riyadh Bassil Abduljabbar\textsuperscript{a,3}

\textsuperscript{a}Dijlah University College, Computer Techniques Engineering Department, Baghdad, Iraq
\textsuperscript{b}Directorate of Industrial Development and Research, Ministry of Science and Technology, Baghdad, Iraq

Email: \textsuperscript{1}hasan.abbas@duc.edu.iq; \textsuperscript{2}mohammed.abdulla@duc.edu.iq; \textsuperscript{3}riyadh.bassil@duc.edu.iq

Abstract— Modern automation robotics have replaced many human workers in industrial factories around the globe. The robotic arms are used for several manufacturing applications, and their responses required optimal control. In this paper, a robust approach of optimal position control for a DC motor in the robotic arm system is proposed. The general component of the automation system is first introduced. The mathematical model and the corresponding transfer functions of a DC motor in the robotic arm system are presented. The investigations of using DC motor in the robotic arm system without controller lead to poor system performance. Therefore, the analysis and design of a Proportional plus Integration plus Divertive (PID) controller is illustrated. The tuning procedure of the PID controller gains is discussed to achieve the best responses of the DC motor. It is found that with the PID controller, the system performance is enhanced, especially in terms of steady-state error but does not provide the required optimal control. The required approach of Ackerman’s formula optimal controller based on state-space feedback is investigated. A GUI using the Matlab environment is created to obtain the DC motor’s responses without using a controller and with controllers. It is found that the proposed approach of the optimal controller has more robustness and enhances the overall performance of the existing PID controller in the form of reducing settling times (from 2.23 second to 0.776 seconds), minimizing percent overshoot (from 27.7 % to 1.31 %) and zero value of steady-state error.

Keywords— DC motor; robotic arm; PID controller; optimal controller; Matlab.

I. INTRODUCTION

Automated technology in the industry field has gained tremendous importance, especially in motion controls [1]. An application of this technology is in the motion of the product, the tool’s trajectory for the cutting process, and the motion of a robotic system [2], [3]. Any efficient manufacturing process requires sufficient energy and torque to put an object in the right venue [4]. The development regarding the performance of any control system leads to complex requirements and accuracy issues. Hard systems can have multiple inputs and outputs. In general, nearly 1960, new technology design for complicated control systems were analyzed and used. These new technologies depend on understanding the state of the system. The situation’s main concept seems to be clear and already existing for many years ago (actually, the type of this case process for a 2-D state space approach). When comparing new and traditional control theories starting from multi I/P multi O/P (MIMO) systems, it can be linear, not linear, and time variable. The last scheme is only applicable for linear systems with a single I/P single O/P (SISO) system. Moreover, new control theory is a time-domain method in which traditional control theory is understood as a harsh frequency band [5], [6].

Robotic process automation is rapidly developed and represents the most advanced technologies across the globe. Robotic arms are vastly used in automation and industrial applications as they perform complicated tasks that are impossible without them [7], [8]. DC motors are widely used for different industrial applications and have many advantages despite required maintenance. They have excellent speed regulation, high starting torque, a wide range of speed, and less complicated in control and drive [9], [10]. The cost of the DC motor is higher than an induction motor. However, some previous studies have investigated DC motors’ speed control were conducted, and various methods have been developed [11-14].

To reduce the effects of load and delay time, the PID controller is used. This controller plays a vital role in feedback systems because of its simplicity, durability, and smoothness. Controllers on a large scale are usually used in the process of industry [15]. PID controller is a type of controllers that can adjust the output. Variable control is
usually based on the errors between specific user-defined points and certain operations measured on variables. However, the PID controller cannot be set in this way because the best response of a control system is achieved by different immobility, loads, and reference speeds [16].

PID controller is compatible with the classical test and error of controlled theory system design and usually does not produce a perfect controlled system [17]. On the other hand, the design of a control system in new control theory allows engineers to achieve optimal controllers relevant to specific responses. Modern control theories enable engineers to include initial conditions in their designs. The design of a control system with conventional control theories’ aid depends on the trial and error approach and usually does not produce a perfect control system. Alternatively, modern control theories allow engineers to design systems with optimal control and best responses [18], [19].

In this paper, a mathematical model of the DC motor used for a robotic arm system has been proposed. The DC motor responses are investigated in three cases: without a controller, with a PID controller, and with the optimal controller using Ackerman’s formula. The proposed work has been programmed using Matlab/ Simulink and GUI design.

II. MATERIAL AND METHOD

A. Robotic Systems

The scheme of the automated system is shown in Figure 1. It includes the robot arm, voltage supply, terminal tools, sensors, front-end computers, software storage, and computer controller. Predefined programs must be considered as a part of the general system. Controlling and programming a robot has a significant effect on its response and the scope of subsequent applications [5], [19].

![Fig. 1 Block diagram of the automation system component](image)

In the automated industrial, there is a need of two essential and contradictory tasks: repetitive and high accuracy. The robotic arm conducts these tasks with high speed and precision. The movement of the robotic arm required a large-scale motor that has optimal control performances. Most robotic arms are equipped with DC motors [20].

B. PID Controller

Today many industrial controllers are used PID controllers because of their simplicity, on-site adjusted, and wide range of applicability, although in some situations, PID controllers may not provide the required optimal control. Control modules in all three terms (P, I, and D) are immensely utilized in industrial systems [16]. The transfer function of the PID controller can be described as in Eq. (1) [21],

\[ K_p + \frac{K_i}{s} + K_d s \frac{K_p s + K_i + K_d s^2}{s} \]  (1)

Where:
- \( K_p \): Relative gain.
- \( K_i \): Integrated gain.
- \( K_d \): Derived gain.

The error signal can represent the input signal to the PID controller, and the controller evaluates both the integral and derivative of the error.

\[ u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt} \]  (2)

The relative gain of a controller (\( K_p \)) can reduce the rising time without affecting the steady-state error. The steady-state error will be increased when the integrated gain (\( K_i \)) decreases. The increase in the derivative gain (\( K_d \)) value will affect the system stability but will reduce the percent of overshooting.

Table 1 shows the impact of PID controller gains on the motor response [22].

<table>
<thead>
<tr>
<th>TABLE 1 GAINS EFFECT OF PID CONTROLLER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
</tr>
<tr>
<td>C/L</td>
</tr>
<tr>
<td>( K_p )</td>
</tr>
<tr>
<td>( K_i )</td>
</tr>
<tr>
<td>( K_d )</td>
</tr>
</tbody>
</table>

Diagrammatically, the PID controller has a block diagram shown in Figure 2 [23].

Adjusting the three parameters of PID (\( K_p \), \( K_i \), and \( K_d \)) is a big problem. Several intelligent techniques are proposed to enhance the tuning of PID gains [24-29]. The following procedure can be used to evaluate the appropriate gain values for the PID controller [30]:

- Choose the actual speed which corresponding to the desired speed, keep the \( K_i \) and \( K_d \) values at constant values and increase \( K_p \) value until the response has sustained oscillations.
- If the system changes then divided the \( K_p \) by 2.
- Increase the KD value and observe the response when increase/decrease the desired speed by 5%. Select KD value which gives a damped response.
- Slowly increase KI value and if the response is sustained oscillations, then divided KI by 2.
- Verify that the response of the controller is satisfied under system requirements.

![Fig. 2 Block diagram of the PID controller](image)
C. Controller Design for State Feedback

An interesting feature of the state space design approach is that the process consists of two separate steps. The first one in which all the states for feedback purposes can be used. Of course, this is generally unrealistic because intern engineers often do not find it necessary to use so many sensors [18]. In classic design, the assumption that all states are available simply let us progress with the first step (control law). The design of the estimator represents the second step, and the status portion is provided by the following states [2,3]:

\[
Y = H X + J U
\]  

(3)

Where:
H: the installed production matrix and
J: the installed transmission matrix.

Combining the estimator and control law results in the final control algorithm, which is based on the estimated state instead of the actual state. This replacement is reasonable, and the combined control law and estimator will provide closed-loop dynamics that are different from those assumed in the separate design of the control law and estimator. The dynamics that are derived from the combined control and estimation laws are called controllers [3].

D. Ackerman’s Formula

The dynamic system described by Eq. (4) depends upon the location of the eigenvalues of \(A\)-matrix; \((\lambda_i)\) for \(i=1, 2, n\) Where \(n\) is the order of the system [2,3].

\[
\dot{X} = AX + BUX(0) = X_0Y = CX
\]  

(4)

The block diagram of the optimal control configuration is shown in Figure 3.

![Block diagram of optimal control configuration](image)

Fig. 3 Block diagram of optimal control configuration

For engineering purposes, such a system has better relocation for the resulting poles (or eigenvalues) in the complex plane, as illustrated in Figure 4.

![Complex plane](image)

Fig. 4 Complex plane

The controllable pair \((A, B)\) is defined by pole placement from \(P\) to \(K\) space.

\[
K^T = K_0^T + P^T \hat{W}^{-1} \hat{X}
\]

\[
K_0^T = e^T A^n
\]  

(5)

Where:
k: control-law gain vector.

\[
w^{-1} = \begin{bmatrix}
e^T \\
e^T A \\
e^T A^{n-1}
\end{bmatrix}
\]  

(6)

The foundation of this mapping is the theorem of pole assignment. For the given polynomial [3,18]:

\[
P(\lambda) = P_0 + P_1 \lambda + \ldots + P_{n-1} \lambda^{n-1} + \lambda^n = [P^T 1] \lambda
\]  

(7)

\[
\lambda = [1 \quad \lambda \quad \ldots \quad \lambda^n]^T
\]  

(8)

An \(n \times n\) array \(A\) and the \(n \times 1\) vector \(B\) similar that the det[\(R\)] is not equal to zero.

\[
R = [B, AB, \ldots, A^{n-1} B]
\]  

(9)

\[
det (\lambda I - A + BK^T) = [P^T 1] \lambda
\]  

(10)

\[
K^T = [P^T 1] E
\]  

(11)

\[
E = \begin{bmatrix}
e^T \\
e^T A \\
e^T A^{n-1}
\end{bmatrix}
\]  

(12)

Where:
e\(T\) is the last row of \(R^{-1}\).

E. Mathematical Model

The automatic arm moves in a horizontal plane by using the DC motor is seen in Figure 5. Assume that the input voltage signal of the system is in a range of zero to ten volts. This signal controls the motor voltage & current. The aim is to design a recoupment strategy such that a voltage from zero to ten volts corresponds to the linear from 0\(^\circ\) degrees of the robotic arm to 90\(^\circ\) degrees. The required response should be as follows: overshoot less than 10\%, settling time less than 0.2 seconds, and the steady-state error is zero.

![DC motor in the robotic arm system of the proposed system](image)
Table 2 gives the parameters of the DC motor used in this study.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Inertia</td>
<td>Kg-m²/sec²</td>
<td>0.01</td>
</tr>
<tr>
<td>b</td>
<td>Damping ratio</td>
<td>N-m sec/rad</td>
<td>0.01</td>
</tr>
<tr>
<td>Rm</td>
<td>The resistance of the DC motor</td>
<td>Ohm</td>
<td>1</td>
</tr>
<tr>
<td>Lm</td>
<td>The inductance of DC motor</td>
<td>H</td>
<td>0.5</td>
</tr>
<tr>
<td>Km</td>
<td>Constant gain</td>
<td>N m/A</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The DC motor transfer function, which equals to the ratio of the robotic arm angle to the input voltage to the motor, is derived, and the final form will be as follows:

$$\frac{\theta(s)}{V(s)} = \frac{k_m}{L_m s^2 + L_m J b R_m s + (R_m b + k_m)^2}$$  \hspace{1cm} (13)

Where:
- $\theta$ is the angle of the robotic arm
- $V$ is the input voltage to the motor

If the parameter values illustrated in Table 1 are compensated in Eq. (13), we get Eq. (14):

$$\frac{\theta(s)}{V(s)} = \frac{0.01}{0.0051 s^2 + 0.0106 s + 0.0011}$$ \hspace{1cm} (14)

### III. RESULTS AND DISCUSSION

The DC motor's responses in the robotic arm are simulated by using Matlab/Simulink software. A complete GUI window has been designed and implemented by which different controllers can be tested for the same plant. This window includes three types: plant without a controller, plant with PID controller, and plant with optimal controller (Ackerman's formula). Each one can be conducted by just click on its command within the GUI window. The view of the proposed GUI window is presented in Figure 6.

![GUI window for the proposed system](image)

In Figure 6, the GUI window includes a graphical plot diagram to view the final response of the selected types with a unit step input. The behavior of the considered system alone, i.e. without a controller, is shown in Figure 7. The corresponding response of the system is very sluggish, and the steady-state error is substantial.

![Step response of the robot arm without a controller](image)

The PID controller gains are evaluated to improve the responses of the motor. These gains are given in Table 3.

<table>
<thead>
<tr>
<th>PID controller gains</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 8 shows the proposed Matlab/Simulink model of the plant and the PID controller.

![Simulink model of the plant with PID controller](image)

The step response of the robotic arm system with a PID controller can be seen in Figure 9. It is noted that the rising time ($t_r$) is 0.514 second, the percent overshoot is 27.7%, peak time ($t_p$) is 1.2 second, the settling time ($t_s$) is 2.23 second and a zero steady-state error.
To calculate the gain vector control (K) using the Ackerman's formula, consider the continuous robotic, open-loop system and control input matrices A and B as:

$$ A = \begin{bmatrix} -2.6574 & 0 \\ 1 & 0 \end{bmatrix} $$

$$ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} $$

$$ C = \begin{bmatrix} 0 & 2.1941 \end{bmatrix} $$

From which the state gain vector (K) can be obtained and given by:

$$ K = \begin{bmatrix} 0.765 \\ 2.325 \end{bmatrix} $$

The regulator eigenvalues can be found by using Eq. (10) and obtained as follows:

$$ S_1 = -2.488 $$

$$ S_2 = -0.935 $$

The DC motor response by using Ackerman’s formula (optimal control) with a step input signal is shown in Figure 10. It was evident that the characteristics of the response are as follows: rising time is 0.508 second, the per cent overshoot is 1.31%, peak time is 1.08 second, the settling time is 0.776 second and a zero steady-state error. Figure 11 shows a step response of the DC motor by using PID and Ackerman’s formula.

From previous results and analysis, a comparison between the responses for different control theories is tabulated in Table 4. Finally, it can be noted that the optimal performance can be achieved by using Ackerman’s formula because of the fast response and zero steady-state error.

![Fig. 10 Step Response of the DC motor by using Ackerman's formula](image)

![Fig. 11 Step response of the DC motor by using PID and Ackerman's formula](image)

### IV. CONCLUSION

This paper presents the optimal control for the DC motor in the robotic arm system using the Matlab environment. It is shown that the performance of the DC motor without a controller is low. Also, there is a vital necessity for the presence of a controller. A GUI using the Matlab environment was built to conduct a comparative investigation for controlling the DC motor in the robotic arm system by using a traditional PID controller and an optimal controller (Ackerman’s formula). The validation results reveal that the optimal controller gives better performance according to the responses of the DC motor in comprehensive range operation including the low overshoot (reduced by 26 %) and short settling time (decreased by 1.454 seconds). Other techniques for optimal control problems can be investigated, which may give more robust and applicable methods such as combining sliding and linear quadratic regulators, intelligent particle swarm optimization, hybrid neural fuzzy controller, and Robust H\(^2\) and H\(^\infty\) optimal control.

### REFERENCES


