Bit Error Rate Performance for Multicarrier Code Division Multiple Access over Generalized $\eta$-$\mu$ Fading Environment

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Abstract—The multicarrier code division multiple access (MC-CDMA) system has received a considerable attention from researchers owing to its great potential in achieving high data rates transmission in wireless communications. Due to the detrimental effects of multipath fading the performance of the system degrades. Similarly, the impact of non-orthogonality of spreading codes can exist and cause interference. This paper addresses the performance of multicarrier code division multiple access system under the influence of frequency selective generalized $\eta$-$\mu$ fading channel and multiple access interference caused by other active users to the desired one. We apply Gaussian approximation technique to analyse the performance of the system. The average bit error rate is derived and expressed in Gauss hypergeometric functions. Maximal ratio combining diversity technique is utilized to alleviate the deleterious effect of multipath fading. We observed that the system performance improves when the parameter $\eta$ increase or decrease in format 1 or format 2 conditions respectively.

Keywords — MC-CDMA, generalized $\eta$-$\mu$ fading, average bit error rate, maximal ratio combining.

I. INTRODUCTION

Multicarrier Code Division Multiple Access (MC-CDMA) is a digital modulation and multicarrier scheme which results from the combination of Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). Based on this combination, three multiple access techniques are formed, viz: Multicarrier (MC) –CDMA, Multicarrier Direct Sequence (DS)-CDMA and Multitone (MT)-CDMA. Therefore, these multiple access schemes have both the advantages and disadvantages of the elements that constitutes them. Hence, these multicarriers techniques are further classified into frequency domain and time domain spreading codes. In case of frequency domain, the original datastream is spreaded using spreading code and modulate different subcarriers with each chip while for time domain, original datastream is serial-parallel conversion and spread using spreading code, then each subcarrier is modulated differently with each of the datastream. The main aim for these multicarrier access schemes is to provide spectral efficiency, interference suppression capability and high data rate transmission over frequency selective fading channels. Similarly, the issue of how to spread the signal bandwidth without increasing the effect of delay spread is also addressed [1]. Hence, the other two multicarrier techniques will not be discussed in this paper.

For the case of MC-CDMA system, it is assumed that the number of subcarriers and the processing gain have to be
the same. Suppose the original symbol rate is high enough to become subject to frequency selective fading, then the signal should first be converted from serial to parallel before spreading over the frequency domain. This is to ensure that multicarrier transmission over each subcarrier should be frequency non selective fading. Therefore, MC-CDMA system transmits the same data bit over all subcarriers without changing the original baud rate [2].

The Bit Error Rate (BER) performance of the MC-CDMA system over frequency selective fading channels was addressed using equal gain combining and maximal ratio combining [3]. Effects of multipath delay on multicarrier received signals was investigated and a new closed form formula for BER was derived based on the orthogonal MC-CDMA system [4].

The authors [5] proposed a simple and efficient receiver design for frame based MC-CDMA in multipath Nakagami-\(m\) fading channels. The performance of the model were illustrated using different fading Nakagami-\(m\) parameters, modulation schemes (4, 16, and 64 QAM) and different number of users. In [6] the system performance is compared with that of the conventional DS-CDMA, it was observed that the multicarrier system outperforms when the system parameters are properly selected. The investigation of the performance of the MC-CDMA system in uplink channel with frequency selective Rayleigh fading channels was conducted [7].

This paper tackles the effects of multipath fading and multiuser interference on performance of MC-CDMA wireless communication system over generalized \(\eta-\mu\) fading channels. The maximal ratio combining diversity method is applied to mitigate the serious impact of multipath fading. The average bit error probability (ABEP) for measuring the performance of the MC-CDMA wireless communication system is derived and expressed in Gaussian hypergeometric functions. The paper is organized as follows: Section II outlines system and channel models. Analysis of the received signal is explained in section III, while the application of the Gaussian approximation method to analyse the signal is described in section IV. In section V, ABEP for the MC-CDMA system is derived, meanwhile the numerical results and discussion is explained in section VI. Finally, section VII deals with conclusion.

II. SYSTEM AND CHANNEL MODEL

A. MC-CDMA Transmitter Model

MC-CDMA system transmits datastream by first carrying out serial to parallel conversion and then spreads datastream in frequency domain (F-domain). The symbol duration of the transmitted signal is \(T_r = qT_b\), where \(q\) is the number of substreams and \(T_b\) is the bit duration before conversion. Hence, after conversion each datastream is spread over \(N_p\) subcarriers using spreading code of \(C_0[0], C_1[1], \ldots, C_{Np-1}\) assigned to user \(k\) in F-domain. The separation between two adjacent subcarriers is assumed to be \(1/T_s\). The spreading codes are assumed to be random variables with values \(-1\) or \(+1\) having equal probability of 0.5. In this case, each chip of data symbol is transmitted in parallel on different subcarrier. Then, the \(k\) th user transmitted signal is given by [8].

\[
S_k(t) = \sum_{i=1}^{N_p} \sum_{j=1}^{N_q} \left[ \frac{2P}{N_p} b^i_j(t) C_k[j-1] \cos(2\pi f_s t + \phi^k_i) \right] \tag{i}
\]

where \(P\) denotes power per bit, \(N_p\) represents the number of subcarriers and \(q\) is the number of bits, \(f_s\) is a subcarrier frequency, \(b^i_j(t)\) is the \(i\) th binary data stream of user \(k\), and \(b^i_j[n] = \sum_{n=-\infty}^{\infty} b^i_j(t)\Delta t\) is a random variable which takes the values of +1 or -1 with equal probability, \(P_{TS}(t)\) represents a rectangular waveform defined as \(p(t) = 1, 0 \leq t \leq \tau\) and zero elsewhere, \(\phi^k_i\) denotes a random phase introduced by carrier modulation assumed to be uniformly distributed in [0,2\(\pi\)].

B. Channel Model

The link between the transmitter and the corresponding receiver is assumed to be slowly flat fading channel for each subcarrier. The fading envelope of the received signal is modelled as generalized \(\eta-\mu\) fading distribution. Therefore, in the analysis of the performance of the system, special cases for the generalized \(\eta-\mu\) distribution are also considered. Hence, the impulse response for the \(k\) th transmitted signal over the \(i\) th subcarrier is given by [9]

\[
h_i^k(t) = \alpha_i^k \delta(t - \tau_k) \exp(-j \psi_i^k) \tag{2}
\]

where \(\alpha_i^k\), \(\tau_k\) and \(\psi_i^k\) denote the attenuation factor, delay and phase shift respectively. \(\delta(.)\) represents Dirac delta function. The delay is assumed to be uniformly distributed over \([0,T_b]\). Therefore, attenuation factor, delay and phase shift are assumed to be constant over a symbol period.

The \(\eta-\mu\) distribution is a general fading distribution that can be used to better represent the small scale variation of the fading signal envelope in a nonline of sight conditions[10]. The considered signal composes of clusters of multipath waves propagating in a nonhomogeneous environment. In a cluster, the phases of the scattered waves are random and have similar delays, with delay-time spreads of different clusters being relatively large [10]. The inphase and quadrature components of the fading signal within each cluster are considered to be independent of one another and have different powers. This distribution includes Nakagami-\(q\), Nakagami-\(m\), Rayleigh and One-Sided Gaussian distribution as special cases. Hence, suppose \(\alpha\) follows generalized \(\eta-\mu\) distribution, then the probability density function of the instantaneous SNR is given by [10], [11]

\[
f(\gamma) = \frac{2\sqrt{\pi} \exp\left(-\frac{2\mu}{\gamma}\right) I_{\mu-0.5}\left(\frac{2\mu H\gamma}{\gamma}\right)}{\Gamma(\mu)H^{\mu-0.5}\gamma^{\mu-0.5}} \tag{3}
\]

where \(\mu = \frac{E^2(\gamma)}{2\text{Var}(\gamma)}\left[1 + \left(\frac{H}{h}\right)^2\right]\), \(E[.]\) is the expectation and \(\text{Var}(.)\) is the variance, \(\bar{\gamma} = E[\gamma]\), \(\Gamma(.)\) is
gamma function and $I_\alpha(.)$ is the modified Bessel function of the first kind of order $x$. The parameters $H$ and $h$ are presented into cases as $H = (\eta^{-1} - \eta)/4$ and $h = (2 + \eta^{-1} + \eta)/4$ where $0 < \eta \leq 1$ is the power ratio of the inphase and quadrature scattered waves in each multipath cluster. The other case is $H = \eta/(1 - \eta^2)$ and $h = 1/(1 - \eta^2)$ where $-1 < \eta < 1$ is the correlation coefficient between the inphase and quadrature waves in each multipath cluster.

C. MC-CDMA Receiver Model

We assume $K$ asynchronous CDMA users simultaneously communicating with the base station employing MC-DMA system. It is also assumed that the users exploit the same number of bits as well as subcarriers. Similarly, perfect power control is assumed. Then, the received signal at the base station is given by [8].

$$r(t) = \sqrt{\frac{2P}{N_p}} \sum_{k=1}^{K} \sum_{j=1}^{q} \sum_{i=1}^{N_{uv}} a_{ij}^k \phi_{ij}^k (t - \tau_k) C_k[j - 1] \times Cos(2\pi f_j t + \phi_{ij}^k) + n(t), \quad (4)$$

where $n(t)$ is the Additive White Gaussian Noise (AWGN) with zero mean and double sided power spectral density (PSD) of $N_0/2$, $\phi_{ij}^k = \phi_{ij} + \psi_{ij}^k - 2\pi f_j \tau_k$, $\psi_{ij}^k$ is a random phase introduced by the channel, $\phi_{ij}^k$ is assumed to be random variable uniformly distributed in $[0,2\pi)$, $\tau_k$ is the misalignment of user $k$ with respect to the reference user, $a_{ij}^k$ denotes the amplitude attenuation due to the channel, it is also a random variable.

III. RECEIVED SIGNAL ANALYSIS

In this case, the receiver makes use of all the received signal energy of an $N_{uv}$ - chip code scattered in the F-Domain. In frequency selective fading channels, different subcarriers can experience amplitude attenuation and phase shift differently. Therefore, this results in destroying orthogonality of the subcarrier which leads to interchannel interference. The statistic decision variable $Z_u$ of the 0th data bit in $u$th data subcarrier for the reference user is given by [8].

$$Z_u = \sum_{i=1}^{N_{uv}} \int r(t) C[v-1] M_{uv} \cos(2\pi f_j t + \phi_{uv}) dt, \quad (5)$$

where $M_{uv}$ is a parameter for identifying diversity combining techniques. Hence, the above equation can be express as

$$Z_u = D + I_1 + I_2 + \eta_0, \quad (6)$$

where $D$ is the reference user or desired user, $I_1$ is a multiuser interference due to subcarriers having the same frequency, $I_2$ is multiple access interference due to other subcarriers with different frequencies. Assume that the received signal $k=1$ is the reference user, $\tau_1 = \tau_0 = 0$ and the phase difference is also zero. Then, the desired user is given by

$$D = \sqrt{\frac{P}{2N_p}} T_s \sum_{u=1}^{N_{uv}} a_{uv} M_{uv} b_u[0], \quad (7)$$

where $b_u[0]$ is the 0th data bit transmitted by $uv$ subcarriers of the reference user. Similarly, the MAI due to the same subcarrier frequency is expressed as

$$I_1 = \sqrt{\frac{P}{2N_p}} \sum_{k=1}^{K} \sum_{j=1}^{q} a_{ij}^k M_{uv} \cos \left[ \frac{1}{2} \left( t - \tau_k \right) C_k[v - 1] C[v - 1] dt, \quad (8)$$

where $\theta = \phi_{ij}^k - \phi_{uv}$, also the MAI due to other active users with distinct subcarriers frequencies is

$$I_2 = \sqrt{\frac{P}{2N_p}} \sum_{k=1}^{K} \sum_{j=1}^{q} \sum_{i=1}^{N_{uv}} \sum_{v=1}^{N_{uv}} a_{ij}^k M_{uv} \cos(2\pi f_j t + \psi_{ij}^k) C_k[v - 1] [j - 1] \times C[v - 1] \cos(2\pi f_j t + \phi_{uv}) dt, \quad (9)$$

and finally

$$\eta_0 = \sum_{i=1}^{N_{uv}} \int C[v - 1] \cos(2\pi f_j t + \phi_{uv}) dt, \quad (10)$$

is noise due to AWGN.

IV. GAUSSIAN APPROXIMATION METHOD

This method is used for determining the bit error rate of the multiple access communication system depending on decision statistic [12]. The interfering components to the desired one are assumed to be zero mean Gaussian random variables. Similarly, AWGN is a Gaussian random process and is a zero mean Gaussian random variable [13].

A. Noise Analysis

The noise engendered by AWGN is described by equation (10). Hence, this noise term is assumed to be Gaussian random variable with zero mean and variance

$$\text{Var} [\eta_0] = \frac{N_{uv} \alpha_{uv}^2}{2E_b}, \quad (11)$$

where $E_b$ is the energy per bit.

B. Interference Analysis ($I_1$)

This multiple access interference (MAI) is caused by the subcarrier having the same frequency. Then MAI is assumed to be Gaussian random variable with zero mean and variance

$$\text{Var} [I_1] = \frac{\alpha_{uv}^2}{3}, \quad (12)$$

where

$$I_1 = \sqrt{\frac{P}{2N_p}} \alpha_{uv}^k M_{uv} \cos \left[ \frac{1}{2} \left( t - \tau_k \right) C_k[v - 1] C[v - 1] dt, \quad (13)$$

C. Interference Analysis ($I_2$)

Similarly, this MAI imposed by user $k$ on desired one is contributed by the other subcarriers signal associated with $i = u, j \neq v$ and is modelled by Gaussian random variables of zero mean and variance [14].
\[ \text{Var}[x] = \frac{\alpha^2}{2\pi \left[(u-v)^2 + (v-u)^2\right]^2}, \quad (13) \]

where
\[ I_2 = \frac{P}{N_p T_s} \sum_{u=1}^{N_p} \sum_{k=1}^{K} b_k^* l_{k,1} c_k^j C_{k-1} [v-1] \]
\[ \times \cos[2\pi (f_{0,1} - f_{u,v}) + \theta] dt \]

Therefore, the decision variable is given by
\[ Z_u = \frac{P}{2N_p T_s} \sum_{v=1}^{N_v} D_{v,u} + \sum_{k=1}^{K} I_1 + \sum_{k=2}^{K} \sum_{k=1}^{N_p} I_2 + \eta_0, \quad (14) \]

The mean of the decision variable is
\[ E[Z_u] = b_u \sum_{v=1}^{N_v} \alpha_{u,v} M_{uv}, \quad (15) \]

and the variance is
\[ \sigma^2 = \frac{N_p N_u}{2F_o} + \frac{K-1}{3} + \frac{(K-1)(N_p-1)qL_m}{N} \sum_{v=1}^{N_v} \alpha_{u,v}^2, \quad (16) \]

where
\[ I_{m} = \frac{1}{q(N_p-1)} \sum_{i=1}^{N_p} \sum_{j=1}^{N_u} I(u,v) \quad \text{and} \]
\[ I(u,v) = \frac{1}{q(N_p-1)} \sum_{i=1}^{N_p} \sum_{j=1}^{N_u} \left[ \frac{1}{2\pi^2 [(u-v)^2 + (v-u)^2]} \right]^2 \]

Hence, if the maximal ratio combining is employed, i.e., \( \alpha_{u,v} = M_{uv} \), then the signal to interference noise ratio is given by
\[ \text{SINR} = \frac{\sum_{v=1}^{N_v} \alpha_{u,v}^2}{\sum_{v=1}^{N_v} \alpha_{u,v}^2 + \gamma}, \quad (17) \]

Where \( \gamma = \left[ \frac{N_p N_u}{2F_o} + \frac{(K-1)}{3} + \frac{(K-1)(N_p-1)qL_m}{N} \right]^{-1} \)

Therefore, the instantaneous signal to interference noise ratio is given by
\[ \gamma = \frac{\sum_{v=1}^{N_v} \alpha_{u,v}^2}{\gamma}, \quad \text{Then, the fading envelope } \alpha \text{ can be modelled} \]
\[ \text{as generalized } \eta-\mu \text{ distribution random variable and assuming multipath intensity profile (MIP) to be uniformly} \]
\[ \text{distributed for the MC-CDMA scheme. Therefore, } \sum_{v=1}^{N_v} \alpha_{u,v}^2 \]
\[ \text{obeys noncentral Chi-square distribution with } 2N_p \text{ degrees of freedom. The pdf of the instantaneous SINR is expressed as} \]
\[ f(\gamma) = \frac{2\sqrt{\pi \mu} \Gamma(\mu + 0.5) \Gamma(\mu + 0.5 - 0.5)}{\Gamma(\mu + 0.5) \Gamma(\mu + 0.5 - 0.5)} \gamma^{\mu - 0.5} \exp \left( -\frac{2\mu \gamma}{\gamma} \right) J_{\mu N_o - 0.5} \left( \frac{2\mu H \gamma}{\gamma} \right) \quad (18) \]

V. BIT ERROR RATE ANALYSIS

Since the transmitted signal is coherently modulated using binary phase shift keying (BPSK) , then, the conditional probability of error is given by \([15, 16]\)
\[ P_b = \mathcal{O} \left( \sqrt{\gamma} \right) \frac{\Gamma(0.5, \gamma)}{2\sqrt{\pi}}. \quad (19) \]

Hence, the unconditional probability of error is given by \([15]\)
\[ P_b(E) = \int_0^{\infty} \mathcal{O} \left( \sqrt{\gamma} \right) f(\gamma) d\gamma. \quad (20) \]

Expressing (18) in series form of modified Bessel function \([11]\)
\[ f(\gamma) = \frac{2\sqrt{\pi \mu} \Gamma(\mu N_o)}{\Gamma(\mu N_o + i + 0.5)} \prod_{i=0}^{N_o} \mu^{2\mu \gamma + 2i} H^{2i} \]
\[ \times \gamma^{2\mu \gamma + 2i - 1} \exp \left( -\frac{2\mu h}{\gamma} \right). \quad (21) \]

This series converges for all values of \( \gamma \) except for \( \gamma = 0 \). The average error probability is given by \([15, 16]\)
\[ P_b(E) = \frac{h^{N_p \mu}}{\Gamma(\mu N_o + i + 0.5)} \prod_{i=0}^{N_o} \mu^{2\mu \gamma + 2i} H^{2i} \]
\[ \times \gamma^{2\mu \gamma + 2i - 1} \exp \left( -\frac{2\mu h}{\gamma} \right). \quad (22) \]

where \( \beta = \frac{2\mu h}{\gamma} \) and \( \text{F}_2 \) denotes Gauss hypergeometric function.

VI. NUMERICAL RESULTS AND DISCUSSION

The BER performance of MC-CDMA system over generalized \( \eta-\mu \) fading channels is carried out using (22). In this case, the number of iterations used for evaluating the series which yields better results is 20 times.

Fig. 1 depicts the BER versus SNR, it is seen that the BER increases as the number of users increase as well. Hence, the system performance degrades. As illustrated in fig. 2, the BER decreases as the parameter \( \eta \) increases resulting to system performance improvement. But in fig. 3 BER decreases when the value of \( \eta \) decreases as well. Therefore, the performance of the system in these two cases format 1 and format 2 improves generally.

In fig. 4 we observed that the BER decreases when the SNR increases. Then the performance of the system improves. Fig. 5 illustrates BER versus number of subcarriers. It is shown that the BER decreases as the number of subcarriers increase and increases when the number of users increase as well. Similarly, fig. 6 shows that the BER in each number of subcarriers increases as the number of users increase too.
MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels ($\mu=1.5$, $\eta=2$, format 1).

Fig. 1 MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels (Users=3, $\mu=1$, Format 1).

Fig. 2 BER versus SNR for MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels (Users=3, $\mu=1$, Format 1).

Fig. 3 BER versus SNR MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels (Users=3, $\mu=1$, Format 2).

Fig. 4 Effect of signal power on MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels.

Fig. 4 Effect of signal power on MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels.

Fig. 5 Effect of active users on MC-CDMA system performance over generalized $\eta$-$\mu$ fading channels.

Fig. 6 Impact of subcarriers on performance of MC-CDMA system over generalized $\eta$-$\mu$ fading channels.

VII. CONCLUSION

The BER performance of the MC-CDMA system over frequency selective generalized $\eta$-$\mu$ fading channel is investigated. Numerical results show that the severity of the
multipath fading can be reduced by increasing the number of the subcarriers. It is observed that the performance of the system improves when SNR increases. Similarly, we deduced that the performance of the system becomes better when the parameter $\eta$ increases and decreases in format 1 and format 2 conditions respectively.

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