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Developing an Artificial Neural Network Algorithm for Generalized Singular Value Decomposition-based Linear Discriminant Analysis

Rolysent K. Paredes^{#1}, Ariel M. Sison^{*}, Ruji P. Medina^{#2}

[#] Graduate Programs, Technological Institute of the Philippines, Quezon City, Philippines, 1109 E-mail: ¹rolysent@gmail.com, ²ruji.medina@tip.edu.ph

* School of Computer Studies, Emilio Aguinaldo College, Manila, Philippines, 1000 E-mail: ariel.sison@eac.edu.ph

Abstract— Artificial Neural Networks (ANN) form a dynamic architecture for machine learning and have attained significant capabilities in various fields. It is a combination of interrelated calculation elements and derives outputs for new inputs after being trained. This study introduced a new mechanism utilizing ANN which was trained using Bayesian Regularization Back Propagation (BRBP) to improve the computational cost problem of the existing algorithm of the Generalized Singular Value Decomposition-based Linear Discriminant Analysis (LDA/GSVD). The proposed approach can minimize the number of iterations and mathematical processes of the existing LDA/GSVD algorithm which suffers time complexity. Through simulation using BLE RSSI Dataset from UCI which has 105 classes and 13 dimensions with 1420 instances, it was found out that ANN improved the computational cost during the classification of the data up to 57.14% while maintaining its accuracy. This new technique is recommended when classifying big data, and for pattern analysis as well.

Keywords— artificial neural network; bayesian regularization back propagation; generalized singular value decomposition; ANN for LDA/GSVD; linear discriminant analysis.

I. INTRODUCTION

Classification is one of data mining's approaches in which it is used to forecast, assign or predict hidden occurrences to their predefined groups [1]. It is mostly employed to evaluate a given dataset, consider each item of it, and allocates this item to a specific group [2]. There are some types of classification algorithms which include ID3, K-Nearest Neighbor (KNN) classifier, C4.5, Naive Bayes, Artificial Neural Network (ANN), Support Vector Machine (SVM), and Linear Discriminant Analysis (LDA) [2, 3].

Thus, it was found out that LDA outperformed other classification models and algorithms [3], and has been utilized widely in the previous years for dimensionality reduction, detection, and supervised learning [4]. It has been extensively employed as well in numerous applications [5-12]. It is supervised learning [13, 14] which intends for ideal conversion directions by reducing usual within-class scatter and make most of the average between-class scatter. Further, LDA has other advantages such as (a) inexpensive application; (b) adapts in discriminating non-linear datasets; and (c) coherence to Bayesian classification [15, 16]. Also, it surpasses PCA concerning classification performance [17].

It has the benefit of looking for projection vector that produces ideal discrimination among different collections of observations [18]. However, it fails when the matrix is singular due to small sample size (SSS) problem [8, 19, 20] which happens if the quantity of the training vectors is less than the dimensions. Because of that, the calculation of eigenvalues and eigenvectors becomes unbearable [4, 8, 20, 21].

To deal with SSS problem on LDA, there were several solutions proposed such as exponential discriminant analysis (EDA) technique which was suggested to solve the undersampled problem [22]. Other proposed method is the Spectral Regression Discriminant Analysis (SRDA) which casts discriminant analysis into a regression framework [23]. The Regularized Discriminant Analysis offers a computation concerning the relationship among the dilemmas of multiclass discriminant analysis and multivariate regression [24]. Also, direct LDA (D-LDA) [25], MBLDA [20], and LDA/OR [26] are for classifying multidimensional data with fast learning capability. The two-stage method utilizing bidirectional LDA and RLDA was designed for twodimensional data only [14] while the split and combined approaches for LDA (SC-LDA) was developed to replace the full eigenvector decomposition [27]. However, the

widely used technique in solving SSS problem is the Generalized Singular Value Decomposition (GSVD) since it can overcome the mathematical problems integral in establishing the scatter matrices [28]. GSVD is also generally applied by various discriminant analysis approaches [21, 29], and a typical method for computing the matrix singular problems in different mathematical solutions [21, 30]. Moreover, GSVD on LDA (LDA/GSVD) provides extraordinary recognition accuracy [31] that is why many researchers used and developed variance of it. However, GSVD suffers from computational cost [12, 31-33] which can cause a longer time in classifying datasets when applied to LDA.

Thus, the purpose of the study is to improve the computational cost of LDA/GSVD by utilizing Artificial Neural Network (ANN). This technique will eliminate the mathematical computations and numerous iterations that are involved in the existing LDA/GSVD algorithm which compromise time complexity making it less efficient. Further, if there is a new instance for classification in the existing LDA/GSVD, the whole process of the algorithm will be repeated from the very start. With the proposed enhanced LDA/GSVD, learning can be done from the datasets and classify each instance, whether new or previously part of the training or testing, will be faster because it will not go back to the start of the whole procedure.

The use of ANN in developing the algorithm has the benefit of accuracy. Also, ANN is equipped with the uniqueness of concurrent processing, can learn and recall data relationships, and mapping of non-linear instances [34, 35]. It is also used in several mathematical calculations [36]. ANNs are applied in several real-world purposes because of their capability concerning resiliency and stableness even in noisy data and for its fault tolerance [37]. Thus, the widely employed method is Back Propagation Neural Network (BPNN). It is composed of input layer, hidden layers, and output layer [38]. An example of BPNN is Bayesian Regularization Back Propagation (BRBP). BRBP offers robust approximation for difficult and noisy inputs. Thus, it works excellently by removing network weights which have no impact on the problem solving and presents improvements on evading the problems of local minima [37]. Furthermore, it delivers weights into a training function while advancing the simplification performance of the old BPNN automatically [38].

Besides, the proposed new approach in this study aims to maintain the existing LDA/GSVD's performance concerning its accuracy and to make its classification and prediction faster. Moreover, this new technique can also be adapted to other mathematical models or computations that implement GSVD.

A simulation of the existing LDA/GSVD algorithm has been presented for comparison with the results of the simulation having the new approach which is the utilization of ANN regarding the computational cost or time complexity, and accuracy.

II. MATERIAL AND METHOD

The study simulates the existing and the enhanced approach for LDA/GSVD algorithms. Dataset from UCI was used during the simulation. This dataset comprises RSSI readings collected from a collection of Bluetooth Low Energy (BLE) iBeacons in a practical indoor setting for navigation and localization applications. There are 105 classes, and 13 dimensions with 1420 instances in the dataset [39].

Table 1 shows parts of the dataset where dimensions are labeled initially from B3001 to B3013 which match to the 13 iBeacons. RSSI values are negative. Thus, more essential RSSI values designate closer adjacency to a certain iBeacon. A value of -200 implies that the iBeacon is very far. For class labeling, the column and row of the iBeacon's position on the map are employed (e.g., I04 means that it is for column I, row 4).

TABLE I PARTS OF BLE RSSI DATASET

Class Label	Dimension Name	RSSI Value	
I04	B3001	-75	
	B3002	-198	
	B3003	-200	
	B3004	-200	
	B3005	-200	
	B3006	-200	
	B3007	-200	
	B3008	-200	
	B3009	-74	
	B3010	-200	
	B3011	-200	
	B3012	-200	
	B3013	-200	
	B3001	-200	
	B3002	-200	
	B3003	-200	
	B3004	-200	
	B3005	-200	
	B3006	-200	
002	B3007	-200	
	B3008	-200	
	B3009	-74	
	B3010	-200	
	B3011	-200	
	B3012	-200	
	B3013	-200	
U01	B3001	-200	
	B3002	-200	
	B3003	-200	
	B3004	-80	
	B3005	-200	
	B3006	-200	
	B3007	-200	
	B3008	-200	
	B3009	-200	
	B3010	-200	
	B3011	-200	
	B3012	-200	
	B3013	-200	

Figure 1 presents the bar graph of the class labels with the corresponding number of instances. It can be observed in figure 1 that there are classes which in some instances are lower than the number of dimensions such as D13, D14, E14,

F08, G15 and few others. Thus, it can lead to SSS problem on the classical LDA that is why LDA/GSVD shall be used.

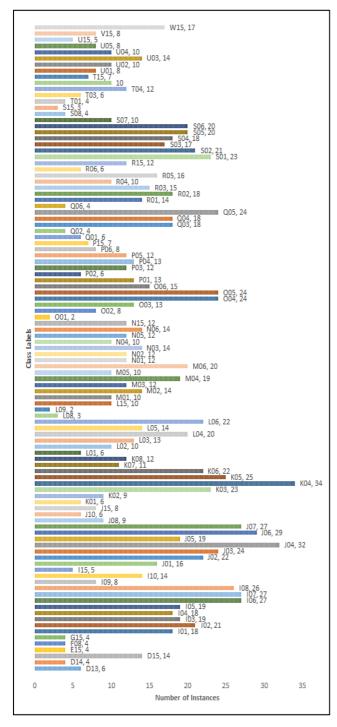


Fig. 1 Classes and number of instances of the BLE RSSI dataset

The study introduced a new approach for LDA/GSVD by utilizing ANN. The tansigmoid transmission function was utilized for the hidden layers' activation function. The flow of the procedure in training the enhanced algorithm is shown in figure 2, and the trained network's architecture is presented in figure 3. The ANN architecture is formed from 13 input variables which are the dimensions, and the corresponding 13 output variables are the expected feature subspaces. For the architecture to learn and predict the possible outcomes, the feature subspaces must be derived from the existing LDA/GSVD algorithm (table 2) since each dimension will have a corresponding feature subspace.

These dimensions and feature subspaces will be used in training and testing. For the sampling, 70% of the instances of the dataset were allocated for training, and 30% for the testing. Moreover, in training of the network, Bayesian Regularization Back Propagation (BRBP) was employed.

TABLE II EXISTING LDA/GSVD ALGORITHM

Algorithm: Existing LDA/GSVD Algorithm

For the matrix $A \in \mathbb{R}^{m \times n}$ with k groups, it calculates the matrix's columns $G \in \mathbb{R}^{m \times (k-1)}$, which maintains the configured cluster dimensionally narrowed space, and determines (k - I)-dimensional depiction Y of A.

Step 1: Calculate $H_w \in \mathbb{R}^{m \times n}$ and $H_b \in \mathbb{R}^{m \times k}$ from A Step 2: Solve the $K = (H_b, H_w)^T \in \mathbb{R}^{(k+n) \times m}$ for its orthogonal decomposition.

$$P^T K Q = \left(\begin{array}{cc} R & 0\\ 0 & 0 \end{array}\right)$$

Step 3: Let $t = \operatorname{rank}(K)$.

Step 4: Calculate W from the SVD of P(1 : k, 1 : t), which is $U^T P(1 : k, 1 : t) W = \Sigma_A$.

Step 5: Solve the first k - 1 columns of

 $X = Q \left(\begin{array}{cc} R^{-1}W & 0 \\ 0 & I \end{array} \right)$

and allocate those to G.

Step 6: $Y = G^T A$.

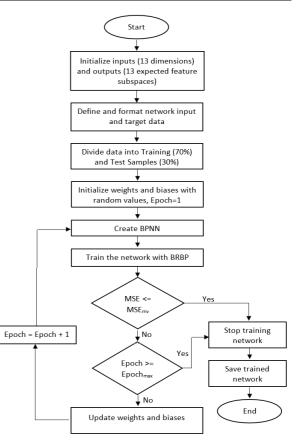


Fig. 2 Training process flowchart of the ANN for LDA/GSVD

After saving the trained network, it will become a module or subroutine that will be used to solve the expected new feature subspaces of the inputs. Thus, the algorithm (table 3) was used to compute the feature subspaces of the instances of the BLE RSSI dataset.

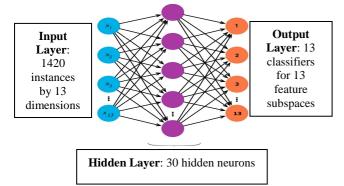
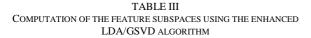


Fig. 3 Artificial Neural Network Architecture for the LDA/GSVD algorithm



- Algorithm: Computation of the Feature Subspaces
- Enter the 13 values of the 13 dimensions.
 Compute the 13 feature subspaces using the module from the
- trained network.

3. Return the computed feature subspaces.

III. RESULT AND DISCUSSION

Using MATLAB R2014a, both algorithms, existing and enhanced LDA/GSVD, were coded and ran on a PC with the processor of Intel® Core i5, 4GB RAM, and 2.7GHz speed.

A. Dataset without LDA/GSVD Classification

Figure 4 below shows the graphical representation of the data without performing LDA/GSVD. Since the dataset is multi-dimensional, for this example, only the first two dimensions are shown in the graph. It can be seen that instead of 105 classes, data points are grouped in approximately three (3) classes. Thus, all of these data points cannot be distinguished as to what classes they belong.

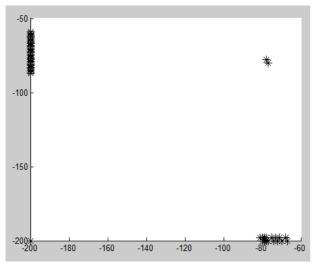


Fig. 4 Graph of the dataset without applying LDA/GSVD

B. Existing LDA/GSVD Algorithm

For classifying 105 classes with 13 dimensions, and a total of 1420 instances, the LDA/GSVD algorithm took 7 seconds to finish. The computational cost was obtained using equation 1.

$$CC = ET - ST \tag{1}$$

Where CC is the computational cost, ET means end time or the time when the program finished to execute all the instructions), and ST means start time or the time when the program starts to execute.

Figure 5 shows the graph for the feature subspaces of the first two dimensions of the dataset using the existing LDA/GSVD. Also, figure 5 shows a better separation of data points compared to figure 4. Due to the number classes and instances, most of the data points with same feature subspaces overlap with each other.

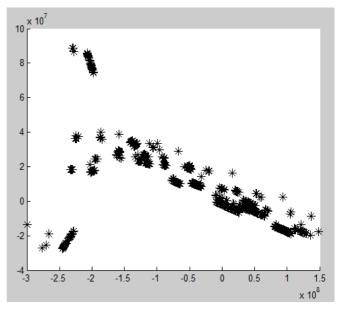


Fig. 5 Graph of the feature subspaces after applying LDA/GSVD

C. Enhanced LDA/GSVD

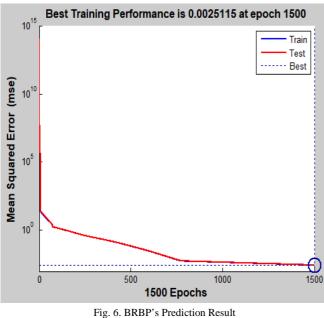
The performance functions were used in the study which includes the Mean Squared Error (MSE) and Regression (R) to evaluate the performance of the ANN for LDA/GSVD algorithm. MSE is the average squared difference between experimental output values and the fed targets in training.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (t_i - a_i)^2$$
(2)

Where *n* is the sample set's size, a_i is the ANN experimental or observed output and t_i is the matching targets. Regression (R) computes the outputs and targets' correlation. When the value of R is 1, it signifies a good or close relationship, otherwise a random relationship [38].

Figure 6 depicts the performance of training and test samples using BRBP algorithm. The graph shows that the test and training samples overlap with each other. Thus, training and test curves continue to stabilize every time the epoch increments. At epoch 1500 the MSE error is approximately 2.5115×10^{-3} . Further, the histogram in figure 7 presents the frequency of the instances per error. The measurement of the error is by subtracting the targets and the

resultant outputs. The most significant error in the training was at around 0.2443.



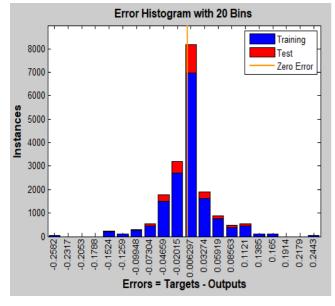


Fig. 7. BRBP's Histogram of Error Sequences

Figure 8 shows BRBP algorithm's correlation. Thus, the graphs present that the algorithm is accurate and better because the MSE is less than zero and the value of R for the training, test, and overall analysis is 1. Further, table 4 shows the performance of the enhanced LDA/GSVD.

TABLE IV PERFORMANCE OF ANN ALGORITHM FOR LDA/GSVD USING BRBP

Dataset Sample	Mean Square Error Regression	
Training	5.6896e-05	1
Testing	6.1412e-05	1

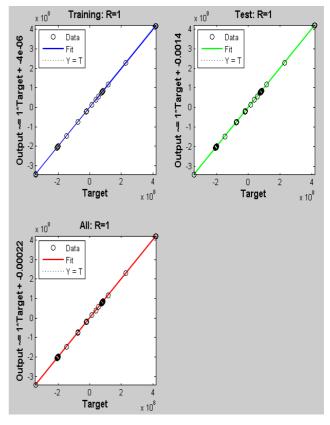


Fig. 8. BRBP's Regression Analysis

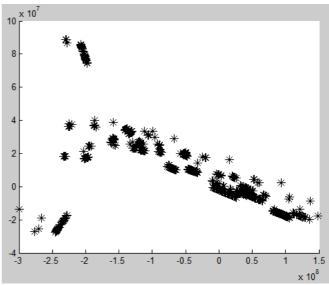


Fig. 9 Graph of the feature subspaces after applying the enhanced LDA/GSVD

It is noticeable that figure 9 above which presents the graph of the feature subspaces using the enhanced algorithm is very much similar to figure 5 which utilized the existing LDA/GSVD. It is a manifestation that the accuracy of the improved LDA/GSVD maintains the accuracy of current LDA/GSVD algorithm.

Using equation 1, Table 5 presents the computational costs of two algorithms. It is evident that the enhanced LDA/GSVD improved the computational cost by 57.14%. The values for the computational costs may be too small because there are only 1420 instances that composed the dataset.

 TABLE V

 COMPUTATIONAL COSTS OF THE EXISTING AND ENHANCED ALGORITHMS

Algorithm	ST ^a	ЕТ ^ь	CC^{c}
Existing LDA/GSVD	08:51:02	08:51:09	7 seconds
Enhanced LDA/GSVD	09:35:43	09:35:46	3 seconds
Improvement of the Enhanced LDA/GSVD			57.14%

a. Start Time, b. End Time, c. Computational Costs

IV. CONCLUSIONS

Simulation results showed that enhanced LDA/GSVD using ANN outperformed the existing LDA/GSVD algorithm regarding computational cost during the classification of the datasets. Thus, it makes the new approach an efficient way of doing LDA/GSVD. It is also evident in the simulation that the new technique using BRBP can obtain the best performance of accuracy by increasing the number of epochs. With that, the new mechanism is highly recommended especially if the dataset has many instances and dimensions due to its lower computational cost. Moreover, implementation of the enhanced LDA/GSVD algorithm to big data will be the next research to be done.

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